# An unbinned test for Quantum Gravity effects in high-energy light-curves.

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*Abstract*. **Some models of quantum gravity can predict observable effects on the propagation of light: most notably an energy dependent dispersion, where the speed of light is seen to vary with the energy of the photon. As quantum gravity effects should appear at the Planck scale they will be very small and so require very high energy photons to travel large distances before even becoming noticeable. Precisely because this effect is greater for the most energetic photons (dt $\sim$ 10 s/TeV/Gpc), ground-based gamma-ray measurements of large AGN flares are the ideal resource for performing such tests. The modest photon flux combined with the fact that these experiments are capable of recording the photon times with great resolution suggests the use of unbinned algorithms as an optimal solution for testing models of quantum gravity. In this paper we discuss the application of a non-parametric test to such datasets, analysing its limitations and exploring the potential benefits.**

*Keywords*: **Lorentz invariance violation, statistical methods, quantum-gravity**

## I. INTRODUCTION

The study of phenomena in which both quantum mechanical and general relativistic effects are important, motivated theoretical efforts to construct a theory capable of describing gravitation at the subatomic level: the so-called Quantum Gravity (QG) theories. One of the most fundamental results, common to several competing approaches to QG, is the quantization of the space-time continuum, which appears in the form of a space-time uncertainty relation $\Delta x \Delta t \geq const$ (e.g., [8]).

A consequence of this discreteness of space-time is that the vacuum will interact with energetic photons on the Planck scale, acting analogously to a medium that absorbs and re-emits radiation by excitation of its internal degrees of freedom [11]. Lorentz invariance violation (LIV) arises in this context due to a modified dispersion relation for the photon, resulting from a non-trivial, spectral dependent refractive index for the vacuum of the form $n - 1 \sim E_\gamma / E_{QG}$ [1], where $E_{QG}$ is the energy-scale for QG, expected to be of the order of the Planck energy $\sim E_P \simeq 10^{19}$ GeV [8].

For photons of energies $E << E_{QG}$, the perturbed dispersion relation can be approximated by a series expansion of the form [5]:

[1]Notice that here, unlike an ordinary medium, the vacuum refractive index *increases* for smaller wavelengths in QG models.

$$c^2 \mathbf{p}^2 = E^2 [1 + \xi E/E_{QG} + O(E^2/E_{QG}^2)] \quad (1)$$

Despite the vanishingly small velocity corrections, of the order of $10^{-15}c$ for a 1 TeV photon, the observation of extragalactic gamma-ray sources such as gamma-ray bursts (GRBs) and active galactic nuclei (AGNs) is a promising laboratory to test this prediction of QG theories. This is because the variations on the speed of light, integrated over the large propagation distances of the photons, result in sizeable delays that could be directly measured by high-accuracy timing experiments [5], which including the cosmological effects of propagation on an expanding universe [7]:

$$\Delta t = H_0^{-1} \frac{\Delta E}{E_{QG}} \int_0^z h^{-1} dz, \quad (2)$$

where $H_0$ and $h$ are respectively the Hubble constant and its associated dimensionless parameter [13], and $z$ is the redshift of the source. In the analysis of broad spectral band light-curves, this delay will manifest as a time-lag between the arrival times of the lowest and the highest energy photons of $\approx 10\xi$ s Gpc$^{-1}$ TeV$^{-1}$.

Traditionally, high-energy experiments have drawn from this principle and, by splitting the light-curves in two or more energy bins, have looked for significant shifts in the times of bursts or sharp features between them, deriving upper limits to the magnitude and energy-scale of the QG effects. In the following section we will briefly review the current status and results of these searches, before proceeding to the presentation of our method.

## II. CURRENT RESULTS OF TIME-LAG MEASUREMENTS

In recent years, several high-energy experiments have begun to perform timing analysis in order to identify energy dependent lags in the light curves of distant sources such as GRBs and AGNs. In principle, the former would be the preferred targets for the study because of their large distances and extremely short burst features, reaching down to sub-second and even millisecond timescales ([12] and [6]). The advent of FERMI brings great prospects to the search for LIV signatures, due to a significant increase in sensitivity. The most constraining GRB results to date come from recent FERMI observations of GRB080916C, and give a robust lower limit of $1.3 \times 10^{18} GeV/c^2$ to the energy scales of QG [10].

Nevertheless, AGN observations with ground-based gamma-ray telescopes carry the advantage of observations at much higher energies, increasing the magnitude of the QG-induced lags one is seeking to tens of seconds. Recent results by HESS [1] and MAGIC [3] provide lower limits for the onset of QG effects of $1.44 \times 10^{18}$ GeV and $0.52 \times 10^{18}$ GeV respectively, in agreement with the newest GRB results. For specialized reviews of these latest results see [9] and [17].

### III. UNBINNED TESTS FOR THE DETECTION OF PHOTON DISPERSION

Given the discrete nature of the high-energy data, tests that exploit the full information content of the light-curves by looking at individual photons are a natural choice to exploit the maximum sensitivity of the experiments. The new method we propose for detecting spectral time-lags in the light-curves of high-energy sources has its fundamental idea drawn from the original approaches of [4] or [16]. It consists of using the linear approximation to the energy-dependent delay given in equation (2) to apply a systematic correction $\tau > 0$ to the arrival times of each individual recorded photon of the form:

$$\Delta t = -\tau E, \qquad (3)$$

so as to *cancel* any putative QG effects on photon propagation. Since the applied correction is to be the exact inverse of the original dispersion, the optimal correction is a direct estimate of the QG energy scale and dispersion magnitude $\xi/E_P$.

The QG signature is asymmetric, always introducing a dispersion to the original burst profile. Therefore the correction $\tau$ assumed to most closely cancel the time-of-flight delays is expected also to return a burst profile which is maximally sharp, according to an appropriately chosen measure, so that the problem is then reduced to that of the maximization of a cost function. In [16], the two proposed cost functions are the Shannon Information (or an alternative information-entropy measure) and the average intra-pulse photon-interval, whereas in [4] the authors sought to maximize the total power of the burst around its maximum.

In any approach of this sort there are two basic assumptions in play, which represent limitations to the method:

i.) all energy-dependent dispersion corrected in the cancelation algorithm is supposed extrinsic to the source and due to QG, since we cannot account for effects intrinsic to the emission process;

ii.) the maximally sharp burst retrieved by the linear correction is assumed to be an accurate representation of the original burst profile.

Whilst (i) is an unavoidable condition in (preferential) non-parametric approaches[2], we propose a method that

avoids (ii), substituting it by a somewhat less arbitrary assumption. As discussed in [16] the problem with this latter condition is that it cannot handle equally well cases where we have to deal with overlapping or asymmetric bursts, in which the maximum sharpness condition may not lead to the correct dispersion cancelation parameter.

### IV. KOLMOGOROV DISTANCE

Given two random variables $X$ and $Y$ in $\Re$, the simplest measure of the *difference* between their probability distributions (pdfs) is the *Kolmogorov distance*, introduced by Kolmogorov in 1933 as a metric for random variables in probability space (see [14] and [18]). For $F_X(x) = \text{prob}(X \leq x)$ and $F_Y(x) = \text{prob}(Y \leq x)$, cumulative distribution functions (cdfs) of $X$ and $Y$ respectively, the Kolmogorov distance is defined as

$$D_K \equiv \sup_{x \in \Re} |F_X(x) - F_Y(x)|, \qquad (4)$$

the maximum vertical distance between the two cdfs.

Given a broad spectral range light-curve with sufficient photon statistics, we can meaningfully bin the data in low and high energy bands, creating two light-curves that should in principle superpose, provided that the high and low energy photons were produced simultaneously at the source, without any intrinsic net delay (same as condition 1 from last paragraph). After propagation, if QG effects are present, the profiles of any given burst in the light curve will differ in the two energy bands, due to the different amount of dispersion of the photons, so that the high energy ones will be more strongly shifted towards later arrival times.

Following [16], we represent the bursts by a normalised probability distribution. We construct a photon cell $x_i = 1/dt_i$ at the place of each photon $i$, where $dt_i$ is the waiting time of each photon and $x_i$ is then indicative of the photon density at each time. We then transform these densities into normalised probability measures by defining $p_n = x_n/\Sigma x_n$ for every cell $n$[3]. Figure 1 illustrates the method.

A natural way to quantify the relative dispersion suffered by the low and high energy components of the burst is to calculate the Kolmogorov distance (K-dist) between the two constructed pdfs. By doing so, we are using the less affected (and usually better sampled) low energy burst profile as a reference for the process of finding the best cancelation parameter to the dispersion of the high-energy light-curve, which is more sensitive to the dispersion.

By applying a simultaneous correction as in equation (5) to the arrival times of each photon in the low and high energy bins, we want to find the optimum correction $\tau^*$ which minimizes the K-dist between the two bursts:

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[2]See [15] for an alternative non-binned approach that try to surpass this limitation by introducing a model-dependent cost function, incorporating properties of the source emission process.

[3]We actually construct the $p_n$s from the $log(x_n)$ to reduce the influence of extremely high-density cells that might arise from statistical fluctuations.
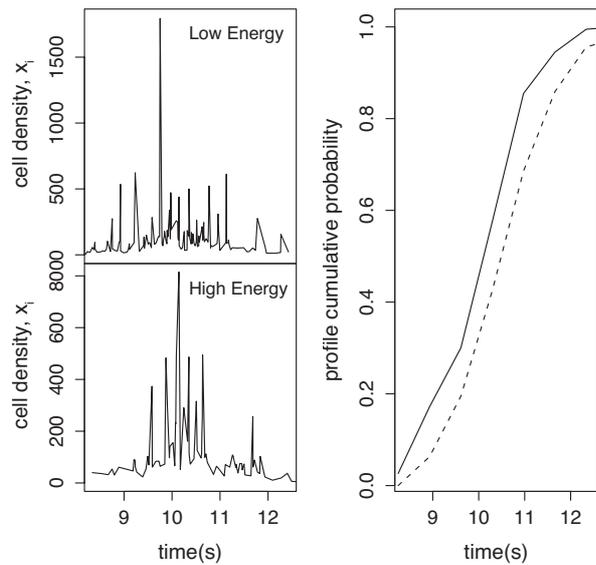
Fig. 1: Illustration of the method of Kolmogorov distances for a Gaussian profile. The panels to the left represent the low- and high-energy cell density representation for the bursts, which appear shifted relative to each other. To the right is the cdf of their correspondent probability distributions; the K-dist is the maximum vertical distance between the curves in this plot.

$$\tau^* : D_{K,\tau^*} = \min_{\tau} \sup_{x \in \Re} |F_X(x) - F_Y(x)|, \qquad (5)$$

corresponding to the QG-induced delay. It is important to note that the cdf is an ideal (and simple) representation to be used for this purpose of comparing two distributions, which acts like a fitting measure of the two profiles as the temporal dispersion is canceled.[4]

## V. PERFORMANCE OF THE METHOD

Following [5], we define a *sensitivity factor*

$$\eta \equiv \frac{\Delta t}{\delta t} \qquad (6)$$

where $\Delta t$ represents the relative delay that two photons of different energies acquired on their travel from the source as in equation (2), and $\delta t$ the width of the burst under study. This parameter measures the power of the method in relation to the size of the light-curve features, which is the most important factor in detecting the delays, and by definition will always be calculated relatively to the delay suffered by a photon of energy equal to the average energy of all events in the burst.

Typical values for the delays are so small ($\sim$ 1-10 s/TeV) that in order to detect their effect we have to

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[4]Notice also that we are *not* relying on confidence intervals and p-values from the Kolmogorov-Smirnov test to compare the distributions, but rather using the K-dist as a metric to differentiate them in probability space.
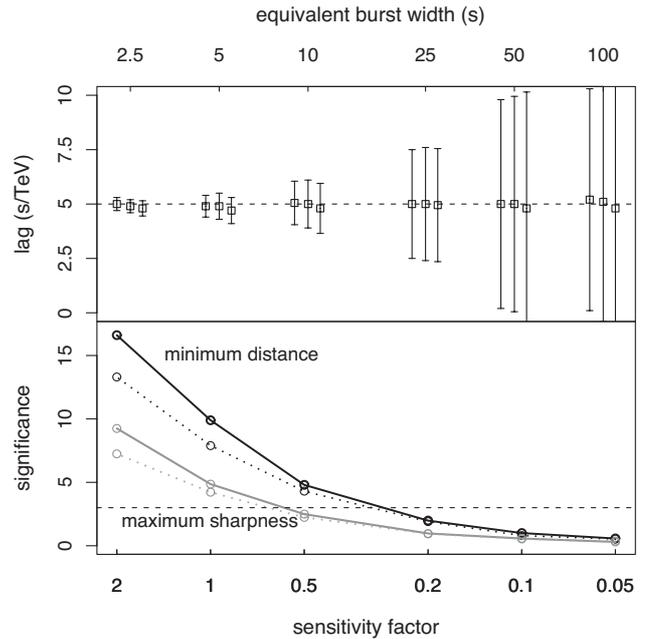


Fig. 2: Sensitivity of the Kolmogorov method. The upper panel shows 10000 MC realisations of the method for detecting lags in function of the sensitivity factor $\eta$. The upper x-axis indicates the equivalent width of the burst in seconds for a source at 500 Mpc and $\xi = 1$, and each tern of points correspond to 0, 10 and 20% error in energy respectively. The bottom panel compares the sensitivity of our method of minimum distance to that of maximum sharpness methods such as in [16] and [3], which have comparable performances among them.

either rely on the observation of extremely short bursts, for which $\eta \geq 1$ [6], or on analysis methods sensitive to small deformations of the burst profile. Apart from GRBs for which $\eta > 0.1\xi$ is frequent, for AGNs detected in the TeV range, the best case to date is from the large flare of PKS 2155-304 in 2006 [2]: its shortest-duration burst of $\sim$ 2 min and average photon energy of $\sim$ 1 TeV, imply a sensitivity factor $\eta \sim 0.05\xi$.

The top panel of figure 2 shows the simulated performance of the K-dist method in function of $\eta$ for a source with spectral index -2.5 at a distance of 500 Mpc. The sensitivity factor is calculated assuming $\xi = 1$. Each simulated burst is a MC realisation of an inhomogeneous poisson process with 1000 events distributed according to a gaussian rate function. The three adjacent points for each value of $\eta$ represent respectively 0, 10 or 20% error in the energy resolution of the observations, which is introduced in the simulation at the moment of the correction for the lag. We can see that the method is only little affected by it, and that the energy error introduces a small systematic underestimation on the size of the lag, but always compatible with the true lag within one standard deviation.

The method as it stands is capable of detecting delays within 3$\sigma$ for $\eta \leq 0.3$ ($\xi = 1$), corresponding to a burst
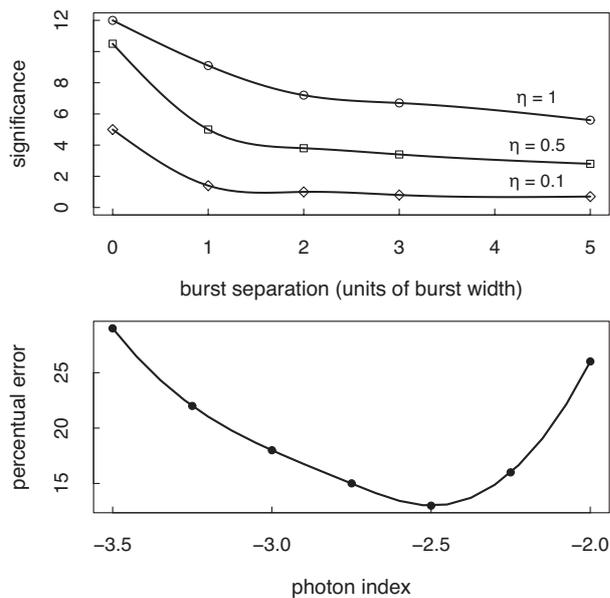
Fig. 3: The top panel shows the variations on the performance of the method for a two-superposed burst configuration; the three curves correspond to different lag-to-burst width ratios $\eta$. The bottom panel shows the variation on the sensitivity of detection of a lag of $\eta = 1$ with the photon index.

of 30 s duration for a source at 500 Mpc. Taking a feature equivalent to the shortest burst in the PKS 2155-304 large flare, we can probe the presence of QG effects in a scale up to $\sim 3 \times 10^{18}$ GeV, a factor of 10 above the Planck energy, and of the order of the most accurate upper limits to date on the QG scale.

Finally, we have tested how the performance of the method depends on the source spectral index and how it varies in the presence of multiple overlayed bursts; the results are presented in figure 3. To test the effect of burst superposition, we generated 4 sets of 10000 MC realisations of a light-curve each, consisting of two identical gaussian bursts separated by 1, 2, 3 and 5 times the individual profile widths, respectively. We can see from the upper plot of figure 3 that the superposition affects the performance of the method as it broadens the *effective width* of the feature in which the lag must be detected. The situation is progressively worsened as the distance between the individual bursts increase, further spreading the combined profile, until both are completely separated (around 3 sigma separation), and the curve reaches its assymptotic limit. At this point, in practice, the individual bursts should be treated individually for better results, which should not represent a problem as long as their separation is comparable or larger than the average photon lag to be tested; otherwise, there could have occurred significant "leakage" of photons between the bursts, which might reflect on the effectiveness of the cancelation method.

The bottom panel of figure 3 shows the sensitivity

curve in function of the photon index of the burst, which presents an optimal minimum around an index of -2.5, for a fixed minimum energy threshold. All bursts generated for testing this effect had the same number of photons (1000) and the energy boundaries of the low- and high-energy profiles are chosen so that the average energy difference between the two profiles and the number of photons in each of them are maximised. The global minimum of the curve at -2.5 results from the fact that for very steep or very hard photon indexes both these factors cannot be ideally optimised.

## VI. CONCLUSIONS AND POSSIBLE VARIATIONS ON THE METHOD

We have exposed here the general accords of an alternative method to test for energy dependent lags in HE light-curves. The method draws inspiration from the unbinned dispersion-cancelation algorithms independently derived by [16] and [3], but it evolves from a maximum sharpness cost-function approach to the minimization of an appropriate distance metric between low- and high-energy components of the burst. By doing so we aim to avoid problems such burst asymmetry that can weaken the assumption of maximum sharpness. In this regard, the metric minimization approach has the role of a dynamic fit between the two components of the profile. We are currently testing ways for further increasing the sensitivity of the method. The search for new, more appropriate, distance measures are also under way and are encouraged to be tested. Applications of this method to VHE and GRB data is underway.

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