# Impact of Chemical Composition of Cosmic Ray on Atmospheric Muon Charge Ratio

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***Abstract*. The MINOS experiment has observed a rise in the underground muon charge ratio $r_\mu$ = $\mu^+/\mu^-$. This ratio can be related to the atmospheric production ratios of $\pi^+/\pi^-$ and $K^+/K^-$. A previous analysis indicates that a simple energy dependent parameterization of the rise in the charge ratio describes both the data and more detailed published Monte Carlo calculations very well. We estimate the size of the previously neglected effect in the parameterisation in this context: the energy dependence of heavy primaries on the $\mu^+/\mu^-$ ratio.**

***Keywords*: Underground Cosmic Ray Muons Muon Charge Ratio Meson Charge Ratio**

## I. Importance of Charge Ratio Measurements

Atmospheric muons come dominantly from the decay of $\pi$s and $K$s produced in hadronic showers when cosmic rays interact in the earth's atmosphere. These muons have been studied with energies ranging from hundreds of MeV to well over a TeV. A quantitative understanding of cosmic ray muons has value for a number of diverse topics, from atmospheric neutrinos to the chemical composition of the highest energy cosmic rays. The charge ratio of cosmic ray muons has been previously measured over three orders of magnitude in energy. Recently the MINOS experiment[1], [2], [8] presented data that for the first time showed a rise in the measured charge ratio

$$r_\mu \equiv \frac{N^{\mu+}}{N^{\mu-}} \tag{1}$$

from previous measurements at high values of $E_\mu$ or more specifically, high values of $E_\mu^{surface}\cos\theta$.

In this paper, we discuss the effect of possible differences in the spectral index of cosmic ray Hydrogen and Helium on the interpretation of the measurement of the muon charge ratio. In particular, we re-fine a simplified model where the rise in the charge ratio can be understood from the properties of $\pi$ and $K$ mesons, and the observation of the rise can be used to determine the $\pi^+/\pi^-$ and $K^+/K^-$ ratios.

Since the primary cosmic rays are mostly positively charged protons, more secondary $\pi^+$ are expected than $\pi^-$. The quark content of the protons and of the atmosphere has been used to estimate the $\pi^+/\pi^-$ ratio to be near 1.27 [3]. The charge ratio for kaons is even higher due to the phenomenon of associated production. Strange particle production starts with the creation of an s quark and an $\bar{s}$ quark. An s quark which ends up in a nucleus is associated with an $\bar{s}$ quark in a $K^+(\bar{s}u)$. The $\bar{s}$ quark will not be in a baryon. There is also $K^+K^-$ pair production. Phase space favors hadronic production of $K^+\Lambda$ over $K^+K^-$ pairs at all energies, so large $K^+/K^-$ ratios are expected.

A standard parametrization of the atmospheric muon energy spectrum is given by Gaisser[4]:

$$\frac{dN_\mu}{dE_\mu} = \frac{0.14E_\mu^{-2.7}}{\text{cm}^2\ \text{s sr GeV}} \times \left\{ \frac{1}{1+\frac{1.1E_\mu\cos\theta}{115GeV}} + \frac{0.054}{1+\frac{1.1E_\mu\cos\theta}{850GeV}} \right\} \tag{2}$$

The two terms inside the curly bracket give the contributions from charged pions and kaons to the muon flux from muons due to $\pi$ and K decay. Above 115 GeV pions tend to interact before they decay and thus it is the relative contribution from kaons that will increase, not by any increased amount of kaon production from primary cosmic ray interactions in the atmosphere.

MINOS has provided the first high statistics measurements of the muon charge ratio at $E_\mu^{surface}\cos\theta > 115$ GeV. In Reference [11] it is shown that this gives the needed sensitivity for extracting information about cosmic ray produced pions and kaons separately. We also note that a similar calculation for neutrinos shows that kaons are the dominant parent for TeV neutrinos, which are the largest backgrounds for astrophysical source searches at neutrino telescopes.

The organization of this paper is as follows: In the following section, we review the model from Reference [11] and [10], a simple energy dependent parameterization of the rise in the charge ratio, before we estimate how a possible difference in the spectral index of the primary cosmic Hydrogen and Helium flux might affect the interpretations. There are two important implications from this model. First, the charge ratio depends not simply on the muon energy $E_\mu^{surface}$, but on the combination of the energy and zenith angle $E_\mu^{surface}\cos\theta$. Second, together with previous measurements at low $E_\mu^{surface}\cos\theta$, observation of the rise in the charge ratio, which mainly occurs between 115 GeV and 850 GeV, can be used to fit the meson production charge ratios $K^+/K^-$ and $\pi^+/\pi^-$.

## II. Model of Pion and Kaon Contributions to the Charge Ratio

We have investigated a generalization of Gaisser's Equation 2 to study separately the positive and negative muon intensities. Using the positive fraction parameters $f_\pi$ and $f_K$, the energy dependency of the positive and negative muons is given by

$$\frac{dN_{\mu^+}}{dE_\mu} = \frac{0.14E_\mu^{-2.7}}{\text{cm}^2\ \text{s sr GeV}} \times \left\{ \frac{f_\pi}{1+\frac{1.1E_\mu\cos\theta}{115\ \text{GeV}}} + \frac{\eta\times f_K}{1+\frac{1.1E_\mu\cos\theta}{850\ \text{GeV}}} \right\} \quad (3)$$

$$\frac{dN_{\mu^-}}{dE_\mu} = \frac{0.14E_\mu^{-2.7}}{\text{cm}^2\ \text{s sr GeV}} \times \left\{ \frac{1-f_\pi}{1+\frac{1.1E_\mu\cos\theta}{115\ \text{GeV}}} + \frac{\eta\times(1-f_K)}{1+\frac{1.1E_\mu\cos\theta}{850\ \text{GeV}}} \right\} \quad (4)$$

where $\epsilon_\pi$ and $\epsilon_K$ have been replaced by their numerical values. We can use equations 3 and 4 to calculate the surface muon charge ratio:

$$r_\mu = \frac{\left\{\frac{f_\pi}{1\ +\ 1.1E_\mu\cos\theta/115\ \text{GeV}} + \frac{\eta\times f_K}{1\ +\ 1.1E_\mu\cos\theta/850\ \text{GeV}}\right\}}{\left\{\frac{1-f_\pi}{1\ +\ 1.1E_\mu\cos\theta/115\ \text{GeV}} + \frac{\eta\times(1-f_K)}{1\ +\ 1.1E_\mu\cos\theta/850\ \text{GeV}}\right\}} \quad (5)$$

The charge ratio of muons from pion decay is $r_\pi = f_\pi/(1-f_\pi)$ and from kaon decay is $r_K = f_K/(1-f_K)$. We will refer to the implications of Equation 5 with energy independent parameters as the "pika" model. Where Feynman scaling is valid, the fraction $x = E_{meson}/E_{proton}$ for $\pi$ and K secondaries does not depend upon $E_{proton}$. Then $f_\pi$ and $f_K$ are also energy independent.

We have used Equation 5 and the measured muon charge ratio to study $r_\pi$ and $r_K$. We have done chi-squared fits in $E_\mu^{surface}\cos\theta$ to the MINOS Near Detector and Far Detector data, and to the L3+C data (*cf.* II). The errors in the two parameters are highly correlated. The latter fit is shown in Figure 3 along with the MINOS (Near and Far) and L3+C data. The agreement between the parametrization and the data is excellent. The parameterization we have used seems sufficient to represent the published data sets.

## III. Effect of Helium on the Charge Ratio

The process addressed in this paper that could affect the energy dependence of $r_\mu$ would be a different spectral index for the heavier cosmic ray primary flux than for the proton flux. This would introduce an energy dependence to the incoming proton to neutron ratio. It is not well established that the energy dependence of the heavy primary intensities is different than that for Hydrogen in the 10 TeV energy range. For this study, we will use Gaisser and Honda's [9] parameterizations of the primary flux as a function primary component k with energy $E_k$, given as

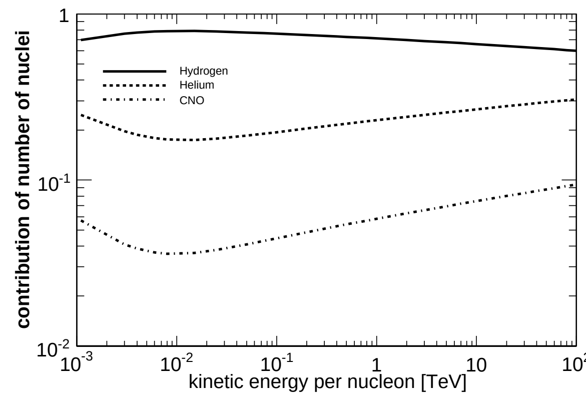


Fig. 1. Fractional contribution of nuclei to the all nucleon spectrum

$$\phi(E_k) = K(E_k/GeV + b\times exp[-c\sqrt{(E_k/GeV)}])^{-\alpha} \quad (6)$$

The parameters in the above equation are given in the Table I below. Note that this reference has a spectral index decreasing slightly with increasing primary mass.

| Parameter | $\alpha$ | $K$ | $b$ | $c$ |
|---|---|---|---|---|
| Hydrogen | 2.74±0.01 | 14900±600 | 2.15 | 0.21 |
| He (A=4) | 2.64±0.01 | 600±30 | 1.25 | 0.14 |
| CNO (A=14) | 2.60±0.07 | 33.2±5 | 0.97 | 0.01 |

TABLE I
Primary flux parameters used in the text.

Figure 1 displays the contributions to the flux as a function of the kinetic energy per nucleon. Based on Monte Carlo calculations using CORSIKA, the mean surface energy of muons which reach 2100 mwe underground is 9% of the primary nucleon energy. The mean fraction of the primary Helium energy, per nucleon, that is transfered to the muon is 11%. We will assume the fraction of the energy transfer for primary carbon, nitrogen, and oxygen to be identical to that of Helium. In the following calculations, the energy profiles of the protons and Helium from Monte Carlo are used instead of these average values, although the effect is not large.

We will calculate the corrections to the fitted $f_\pi$ and $f_K$ parameters in Equation 5 to account for the possible energy dependency of the incoming proton to neutron ratio (due to the Helium and CNO flux). We use a model to estimate the impact of both pion and kaon production by protons and neutrons. We assume two contributions for meson production: pair production of equal number of positive and negative mesons, and leading particle production using quark counting to estimate the excess. This calculation will use meson charge ratios near the previous fit values. We start with $r_\pi = 1.25$ and $r_K = 2.61$ and see how much they change due to the different spectral index for H and He.

First we will model the change in the muon charge

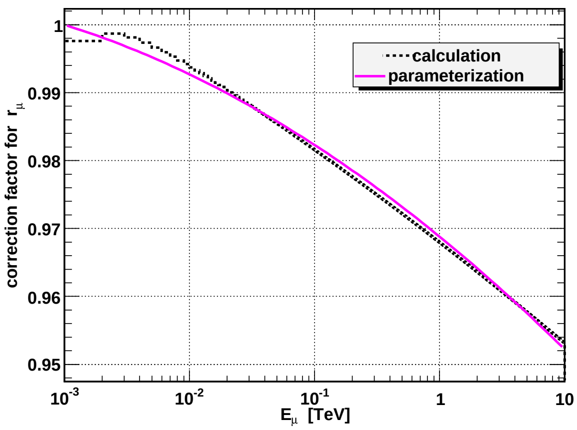


Fig. 2. Correction factor for $r_\mu$ assuming a different energy spectrum for Helium from pion production only.

ratio due to just pion decay when Helium and CNO are added to the primary proton flux. Using CORSIKA separately for cosmic ray protons and Helium of energy $E_{CR}$, we have calculated weights $w^i$ in 11 bins of the ratio $E_\mu/E_{CR}$. In a simple quark model the leading proton and neutron particle in the forward direction will give rise to a charge ratio of 1.00 for symmetric nuclei A-Z=Z.

Setting the muon charge ratio equal to 1.25 at 1 GeV and fixing the normalization of the Hydrogen flux to be 1.317 (in order to get back the charge ratio of 1.25 at 1 GeV), one has for the muon charge ratio

$$r^\pi(E_\mu) = \sum_{i=0}^{10} \Big[ 0.5 \times (w_p^i + w_{He}^i) \times \tag{7}$$

$$\frac{w_p^i 1.317 \times \phi_p(E_{CR}) + w_{He}^i 1.0 \times 4\phi_{He}(E_{CR}) + w_{He}^i 1.0 \times 14\phi_{CNO}(E_{CR})}{w_p^i \phi_p(E_{CR}) + w_{He}^i 4\phi_{He}(E_{CR}) + w_{He}^i 14\phi_{CNO}(E_{CR})} \Big]$$

The 1.0 values are due to the equal production rates of leading $\pi^+$ and $\pi^-$ in our quark counting model.

A subtlety of our notation is that the $\pi$ (and K) superscript represents the energy dependent correction to the charge ratio arising from the different possible spectral index of heavy cosmic ray primaries, while the subscript represents the assumed energy independent meson ratio. Figure 2 displays the corresponding fractional change in the charge ratio as a function of the muon surface energy. Note that the effect is about a 3 percent reduction at 1 TeV. This dependency can be fit to a polynomial in log(E), also shown in Figure 2, with the result

$$r^\pi(E_\mu)/1.25 = \tag{8}$$

$$1 - 0.00575 \times log_{10}(E_\mu/GeV) - 0.00155 \times log^2{}_{10}(E_\mu/GeV).$$

Next we include the contribution from kaon production by the primaries. The cross section for associated production of $\Lambda K^+$ is much larger than that for $K^+K^-$ pairs for both incident protons and neutrons. Again assuming a quark model for the leading kaon in the forward direction from incident protons and neutrons, one only gets leading $K^+$, but twice as many from protons than from neutrons corresponding to their number of $u$ quarks.

$$p(uud) + air \rightarrow K^+(u\bar{s}) + \cdots \tag{9}$$

and

$$n(ddu) + air \rightarrow K^+(u\bar{s}) + \cdots \tag{10}$$

In this model, the positive kaon excess originating from protons ($r_K - 1$) is twice as large as the excess originating from neutrons. For symmetric nuclei with A-Z=Z this leads to a charge ratio

$$1 + \frac{[1 + 1/2]}{2}(r_K - 1) = 1/4 + 3r_K/4 \tag{11}$$

We further assume that the muon charge ratio at 1 TeV is 1.374 (near the MINOS value). This then yields

$$r^K(E_\mu) = \sum_{i=0}^{10} \Big[ 0.5 \times (w_p^i + w_{He}^i) \times \tag{12}$$

$$\frac{w_p^i 2.7 \times \phi_p(E_{CR}) + w_{He}^i 2.275 \times 4\phi_{He}(E_{CR}) + w_{He}^i 2.275 \times 14\phi_{CNO}(E_{CR})}{w_p^i \phi_p(E_{CR}) + w_{He}^i 4\phi_{He}(E_{CR}) + w_{He}^i 14\phi_{CNO}(E_{CR})} \Big]$$

The value 2.275 is due to Equation 11. The leading particle positive excess dominates over $K^+K^-$ pair production, unlike in the pion case. Again we parameterize the dependency in $log_{10}E_\mu$ from Equation 8 such that

$$(1 - r^K(E_\mu)/2.61) = 2/3 \times (1 - r^\pi(E_\mu)/1.25) \tag{13}$$

The effect on the muon charge ratio due to the heavy ions requires that the pion and kaon fractions be modified as follows:

$$f_\pi^*(E_\mu) = 1/(1 + 1/r_\pi \times r^\pi(E_\mu)) \tag{14}$$

$$f_K^*(E_\mu) = 1/(1 + 1/r_K \times r^K(E_\mu)) \tag{15}$$

Including these contributions from heavy primaries does have an impact on the calculation of parameters of the pika model. The charge ratio no longer depends just upon $E_\mu \cos\theta$. While this effect is small it is certainly present and can be accounted for in modeling. Figure 3 shows the effect of the heavy primaries on the two parameters of the pika model. The fit to the data is almost indistinguishable from the previous fit. Note that an increasing heavy primary fraction at high energy will decrease the charge ratio, so to fit to the high MINOS points, a larger value of $r_K$ is required. It is clear that simulations of the muon charge ratio need to include the possible different energy dependence of heavy primaries.

We repeat that the choice of spectral index in Table I is to illustrate the size of a possible effect. If it turns out that the spectral index is independent of chemical composition, the effect described in this paper will not exist.

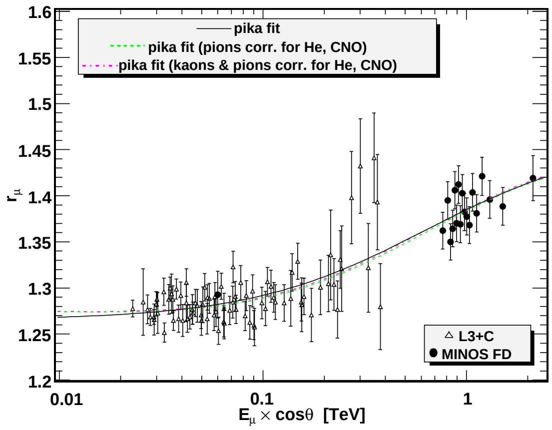


Fig. 3. Fit to the meson charge ratios modifying the pika model with the energy dependence of Helium and CNO production.

## IV. Discussion

We have considered the change in the charge ratio from the fact that $\mu^+$ lose slightly more energy than $\mu^-$ while penetrating the overburden of an underground detector, both for ionization and catastrophic energy loss. This effect was mentioned in Reference [1] but not used to correct the reported charge ratio there. In this paper, we correct the MINOS data to slightly higher values. For the Far Detector data in Reference [1], this is a + 0.6% correction at 2000 mwe detailed reported in References [6] [7], and for the near detector at 250 mwe the correction is +0.37%. We estimate a +0.29% correction for L3+C at 80 mwe.

The central result of this paper is to show the effect on the charge ratio assuming that there is a spectral index for cosmic ray Helium nuclei which is different than the spectral index for protons. Using an index of 2.74 (2.64) for H (He), we get a change in $r_K$ from 2.3 to 2.7 compared to an analysis where the spectral indices were the same. This result was derived by the development and application of Equation 5 and the consequent relationship of $r_\mu$ to $r_\pi$ and $r_K$. We have used that equation to study its consistency with fits to data. In Table II, we show the fits for our parameters to the data reported in References [1], [8] and [5].

| | $r_K$ | $r_\pi$ |
|---|---|---|
| MINOS and L3 | 2.34 ± 0.07 | 1.227 ± 0.003 |
| MINOS N+F | 2.26 ± 0.29 | 1.241 ± 0.035 |
| Helium $\gamma$ 2.74 → 2.64 | 2.73 ± 0.09 | 1.234 ± 0.003 |

TABLE II
PIKA FITS TO $r_K$ AND $r_\pi$ FOR DATA.

It is important to note that the systematic effects considered in this paper, which include remaining errors from randomization, corrections for differences in dE/dx for $\mu^+$ and $\mu^-$ and a possible lower spectral index for Helium, would all tend to raise the fitted values of $r_K$.

Several effects which have not been explicitly considered here are expected to be small. These include the production of muons from charm and other heavy particles, components of the cosmic rays heavier than CNO and possible differences in their spectra, and a variety of scaling violations which would have the effect of making $f_\pi$ and $f_K$ energy dependent. These effects need full simulations to evaluate fully; but it is important that full simulations yield the experimentally measured average values of these parameters. In that context, the analysis presented in this paper can be useful.

## V. Summary

If the spectral index for cosmic ray Helium nuclei is different than the spectral index for protons, e.g. using an index of 2.74 (2.64) for H (He), then we get a change in $r_K$ from 2.3 to 2.7 compared to an analysis where the spectral indices were the same. This result was derived by the refinement of Equation 5 and fits to data applying the consequent relationship of $r_\mu$ to $r_\pi$ and $r_K$. The charge ratio would then no longer depend just upon $E_\mu \cos\theta$. Although the effect is small, it should be accounted for in modeling and searched for in future precision data of $r_\mu$ from new underground experiments. This in turn could give rise to more evidence for a different spectral index for cosmic ray Helium nuclei, independent of the very difficult airborne measurements.

## VI. Acknowledgments

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