The evolution of the moments of the X_{max} distribution with energy and primary particle mass

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***Abstract*. The evolution of the moments of the X_{max} distribution is an important parameter in the study of the primary composition of cosmic rays. In this paper we investigate the evolution of these moments as a function of energy and primary particle mass. We compare different full simulation to semi-analytical models. A numerical expression is proposed in order to fit the evolution of the mean and RMS of the X_{max} distribution as a function of energy and primary particle mass.**

***Keywords*: X_{max} Distribution, Composition and Shower Simulation.**

I. INTRODUCTION

The mass composition of cosmic rays with energy above 10^{17} eV plays a key hole in the high energy astroparticle physics scenario. On the other hand, due to model discrepancies and statistical fluctuations, it is one of the most hard information to be extracted from the data. Mainly two parameters are used in composition studies: a) muon component of the shower and b) depth in which the shower reaches the maximum number of particles X_{max}. For recent experimental results of the use of muon component in composition studies see reference [1] and for results using X_{max} see [2].

In this article we investigate the evolution of the first (mean - $< X_{max} >$) and second (RMS = σ_{Xmax}) moments of the X_{max} distributions using Monte Carlo simulations. The evolution of the mean has been previous studied as a function of energy in references [3], [4], [5]. However its dependency on the primary mass has not deserved the same attention. At the same way, the RMS of the distribution has not been largely studied with Monte Carlo simulations despite the fact that it holds an independent and unbiased information of the primary composition. In this work we focus on the RMS of the X_{max} distribution and on the dependence of the moments on the primary mass.

Monte Carlo simulations of the X_{max} distribution are used in the composition studies in order to test the hadronic interaction models, to indicate the mean mass of the measured showers and to translate astrophysical models into measured X_{max} distribution. Each astrophysical model of particle acceleration delivers a different primary composition that can be tested against the data. The confrontation between astrophysical models and data is done by converting the predicted particle abundance into corresponding X_{max} distributions [5], [6].

In order to perform the conversion between primary abundance and X_{max} distribution large number of Monte Carlo events with a wide range of primary energy and particle type is usually needed. Therefore a parametrization of the X_{max} distribution moments as a function of the main parameters would easy this study by eliminating the need of time consuming simulations. Recently a parametrization of the mean X_{max} distribution was proposed in reference [7]. In this work we perform a similar study and goes beyond by analyzing also the RMS of the X_{max} distribution. At the same time, we propose a simpler parametrization for the $< X_{max} >$ evolution with mass and energy.

II. ANALYTICAL MODEL

It has been shown in reference [3] that an extension of the Heitler model for hadronic showers describes the $< X_{max} >$ evolution with energy up to 10^{15} eV. In this model, the proton $< X^P_{max} >$ as a function of energy is given by:

$$< X^P_{max} > \; = \; X_0 \; - \; 58 \; \log \frac{E_0}{1PeV} \qquad (1)$$

where X_0 is the first interaction depth for protons that can be approximated by $X_0 \approx \lambda_r \ln 2 = 25.6 \; g/cm^2$. In which we have used $\lambda_r = 37 \; g/cm^2$ as the radiation length in air.

Also according to this model the $< X_{max} >$ of a shower started by a particle with mass *A* is:

$$< X^A_{max} > \; = \; < X^P_{max} > \; - \; \lambda_r \ln A \qquad (2)$$

where the superscript *P* stands for proton.

Therefore according to this model, the evolution of the $< X_{max} >$ with energy and primary mass follows a logarithmic law. Some limitations of this model have been studied in the references [3], [8]. Namely the hadronic inelasticity is not properly taken into account and alternative correction have been proposed [3], [8]. Nevertheless, the correction have not been enough either to fully describe the phenomena nor to erase the general logarithmic behavior pointed above.

Recently an numerical model has been proposed to describe the evolution of $< X_{max} >$ with energy and primary mass [7]. In this work a fit of the fully simulation results is done and the following equation is proposed:

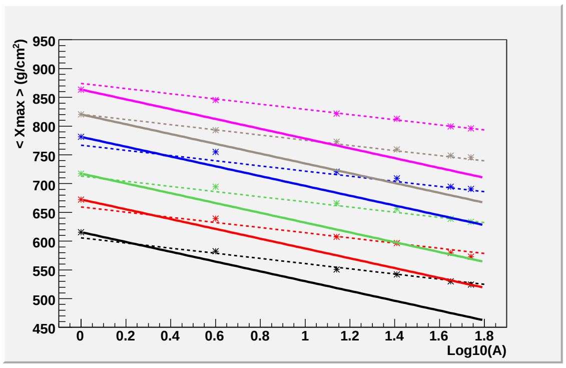

Fig. 1: Mean X_{max} as a function of primary particle mass. Colors represents different energies (black = 10^{16} eV, red = 10^{17} eV, green = 10^{18} eV, blue = 10^{19} eV, brown = 10^{20} eV and magenta = 10^{21} eV.). Full lines represent the analytical model and dashed lines a fit. Asterisks show the values simulated with Sibyll 2.1.

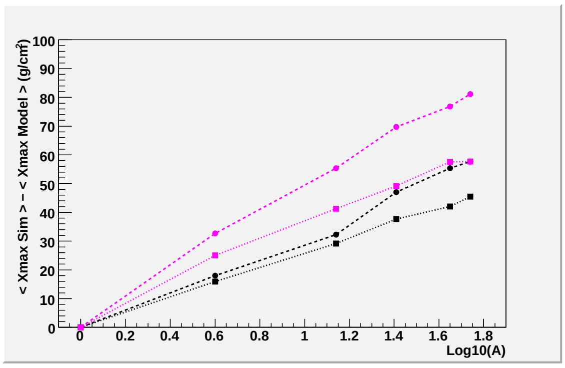

Fig. 2: Difference between the mean X_{max} calculated with the analytical model above and the full simulations. Colors represents different energies (black = 10^{16} and magenta = 10^{21} eV.). Squares represents Sibyll 2.1 and circles QGSJet I.

$$\langle X_{max}\rangle(E,A,i) = \left[a_i\left(Log\frac{E}{A\epsilon}\right)^2 + b_i Log\frac{E}{A\epsilon} + c_i\right] \times(1+\alpha_i A)Log\frac{E}{A\epsilon} + p_i(1+\beta_i A) \quad (3)$$

Figure 1 shows the comparison of analytical model above with full simulations as a function of the primary mass for different primary energies. The colors in the plot represents energies (black = 10^{16} eV, red = 10^{17} eV, green = 10^{18} eV, blue = 10^{19} eV, brown = 10^{20} eV and magenta = 10^{21} eV.). The asterisks show the results of full simulations. The full lines are calculations of the semi-analytical model above (equation 2). The dashed lines are fits to the full simulation which are going to be discussed below. The model proposed in reference [7] would give a similar result to the dashed-lines in agreement with the full simulations.

III. Simulation

The simulations used in this work were done with CONEX v2r2.0 [9], [10] with QGSJet-I [11] and Sibyll 2.1 [12] as the high energy hadronic interaction models. We have simulated 200 showers for each primary energy, primary particle type and hadronic interaction model. We have simulated shower initiated by particles with mass equal to 1, 4, 14, 26, 45 and 56, corresponding respectively to the mean mass of H, He, CNO, Ne-S, Cl-Mn and Fe. The simulated primary shower energy ranges from 10^{16} to 10^{21} eV.

Despite the number of simulated shower for each case is not very high we understand it is already enough to keep the fluctuations below a few percents. More showers are under production and new results including more hadronic interaction models are going to be presented soon.

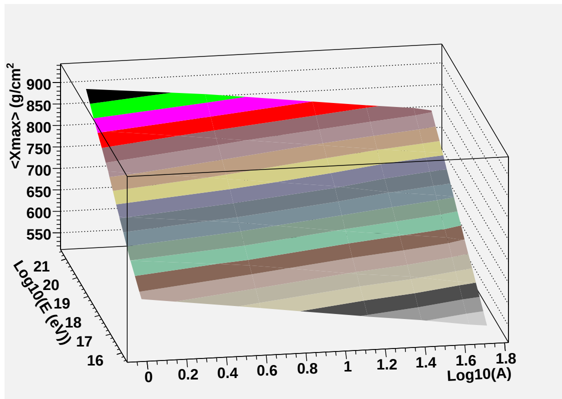

Fig. 3: Mean X_{max} as a function of energy and particle mass. Colors shows intervals of equal X_{max} values.

Figure 2 shows the difference between the analytical model and the prediction of the simulations for two energies 10^{16} (black) and 10^{21} (magenta) eV. Squares represents Sibyll 2.1 and circles QGSJet I.

Figure 3 and 4 show the results of the simulations done with Sibyll 2.1. Similar plots were done for QGSJet-I. The plots show the dependence of the $< X_{max} >$ and $\sigma_{X_{max}}$ with primary energy and particle mass. The simulated points are not shown. We chose to show the smoothed surface in order to emphasize the general trend along both parameters.

Figure 5 shows a comparison between Sibyll 2.1 and QGSJet-I as a function of primary mass. The color code represents energy as said above. Squares represents Sibyll 2.1 and circles QGSJet I. The lines are fits to the points. The general linear dependence with log10(A) is seen for both models. In the energy range from 10^{16} (black) to 10^{19} (red) eV the slope are very similar for Sibyll 2.1 and QGSJet-I. Above 10^{19} a change in slope is seen.

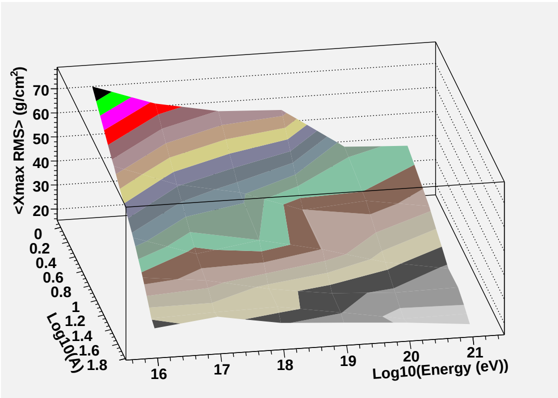

Fig. 4: RMS of the X_{max} distribution as a function of energy and particle mass. Colors shows intervals of equal σ_{Xmax} values.

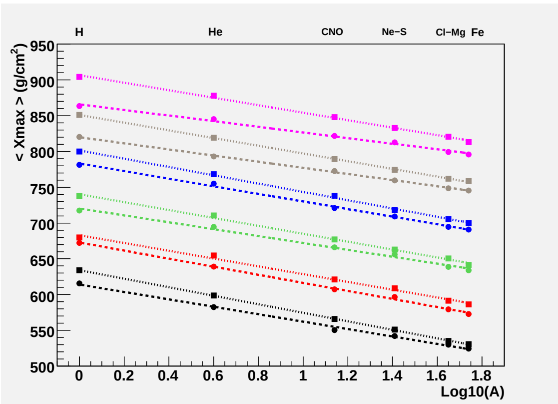

Fig. 5: Mean X_{max} a function of particle mass. Colors represents different primary particles (black = H (A = 1), red = He (A = 4), green = CNO (A = 14), blue = Ne-S (A = 26), brown = Cl-Mn (A = 45) and magenta = Fe (A = 56). Squares represents Sibyll 2.1 and circles QGSJet I.

IV. RESULTS

Figure 3 shows a very smooth dependence of $< X_{max} >$ with energy and primary particle mass. Therefore we have fit this surface with a simple plane equation given by:

$$< X_{max} >= C_1 \log E_0 + C_2 \log A + C_3 \qquad (4)$$

The results of this fit for Sibyll 2.1 and QGSJet-I are show in table I. The result of these fit as a function of particle mass is seen in figure 1 as the dashed lines when we have used Sibyll 2.1. A very good agreement is seen and similar results have been seen for QGSJet-I. The result of the fit as a function of energy is seen in figure 6. In this plot colors represent different primary particles.

Figure 4, on the other hand, does not show a very smooth dependence of $\sigma Xmax$ with energy and primary

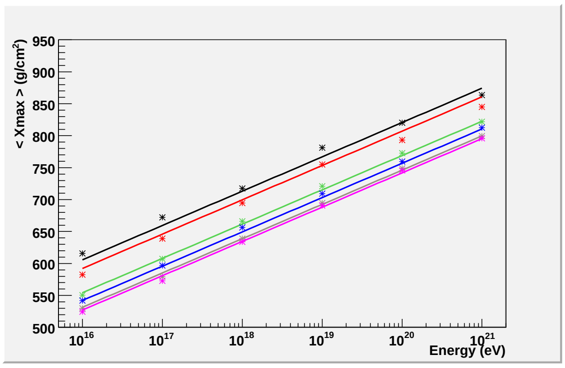

Fig. 6: Mean X_{max} as a function of energy. Colors represents different primary particles (black = H (A = 1), red = He (A = 4), green = CNO (A = 14), blue = Ne-S (A = 26), brown = Cl-Mn (A = 45) and magenta = Fe (A = 56). Full lines show the results of the fit.

particle mass. The projection of $\sigma Xmax$ on to the energy axis shows a small dependence of the slope on primary mass. At the same way, the projection of $\sigma Xmax$ on to the mass axis shows a small dependence of the slope on energy. See the points in figure 7 and 8. Therefore we have fit $\sigma Xmax$ as a function of energy and mass with the following equation:

$$\begin{aligned} \sigma Xmax &= (C_4 + C_5 \log E_0) \times \log A \\ &+ (C_6 + C_7 \log A) \times \log E_0 \\ &+ C_8 \end{aligned} \qquad (5)$$

The results of this fit for Sibyll 2.1 and QGSJet-I are show in table I. The result of these fit as a function of particle mass is seen in figure 7 we have used Sibyll 2.1. The color code has the same meaning explained above. A very good agreement between the simulation (points) and the fit (full lines). Slight worse agreement was seen for QGSJet-I.

The result of the fit as a function of energy is seen in figure 8 we have used Sibyll 2.1. The color code here represents the primary particle type (black = H (A = 1), red = He (A = 4), green = CNO (A = 14), blue = Ne-S (A = 26), brown = Cl-Mn (A = 45) and magenta = Fe (A = 56). A reasonable agreement between the simulation (points) and the fit (full lines). The same level of agreement was seen for QGSJet-I. For low mass primaries a departure from the linear behavior is seen.

V. CONCLUSIONS

This work is the first try in order to fit the moments of the X_{max} distribution as a function of energy and primary particle type. Equations 4 and 5 were fit and the parameters are shown in table I. The parameters can be used in order to calculate the mean and RMS of the X_{max} distribution for any primary particle and energy.

The results represented here make it easier to transform the abundance predicted by astrophysical models

TABLE I: Parameters fit to equations 4 and 5.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
QGSJet-I	-252	-45	53	139	-59	1.0	-4.4	1.0
Sibyll 2.1	-276	-51	56	170	-64	1.3	-6.3	1.3

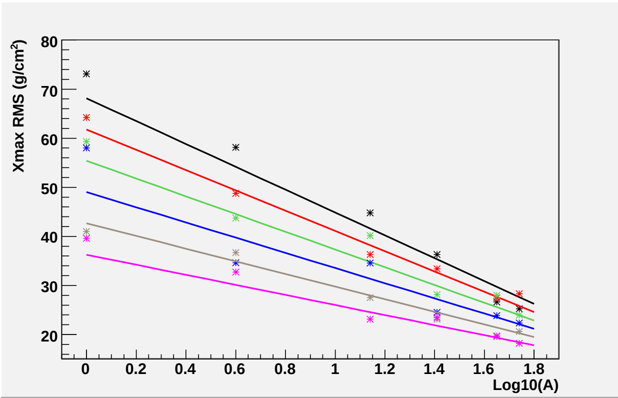

Fig. 7: RMS of the X_{max} distribution as a function of primary particle mass. Colors represents different primary energy (black = 10^{16} eV, red = 10^{17} eV, green = 10^{18} eV, blue = 10^{19} eV, brown = 10^{20} eV and magenta = 10^{21} eV.). Asterisks shows the simulations and full line the results of fitting equation 4 to the points.

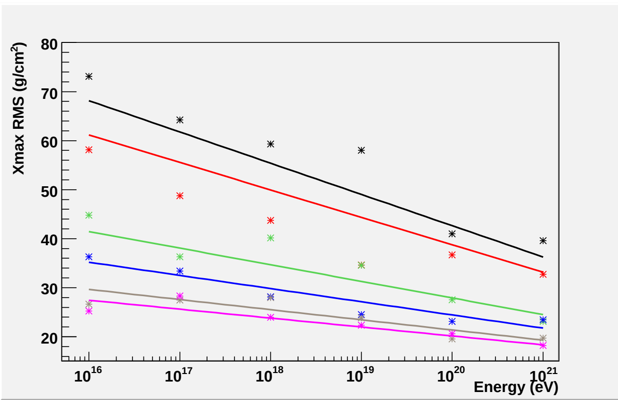

Fig. 8: RMS of the X_{max} distribution as a function of primary energy. Colors represents the primary particle mass (black = H (A = 1), red = He (A = 4), green = CNO (A = 14), blue = Ne-S (A = 26), brown = Cl-Mn (A = 45) and magenta = Fe (A = 56)). Asterisks shows the simulation and full line the results of fitting equation 5 to the points.

into the moments of the X_{max} distribution by eliminating the need of time consuming simulations. It is noticeable that both the mean and the RMS of X_{max} distribution can be fit by a rather simple model for the mass ranging from H to Fe and the energy ranging from 10^{16} to 10^{21} eV.

In a future work we intend to extend these model to other hadronic interaction models. In doing so it might be needed to use more complicated models as the one shown in equation 3. Specially for the EPOS hadronic interaction model for which the dependence on the mean X_{max} with energy is different from the models used here.

VI. Acknowledgments

The author thanks the Brazilian population for supporting his work via FAPESP (Grant: 2008/04259-0).

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