***Abstract*. Astrophysical neutrinos may be produced by several mechanisms, which give different flavor ratios. They oscillate into different flavor ratios during propargations from sources to the Earth. Determination of neutrino flavor ratio is a big challenge to the current neutrino experiments. We show that the neutrino flavor ratio at the astrophysical source can be determined by future neutrino-telescope measurements and the knowledge of neutrino mixing angles obtained from terrestrial experiments.It is demonstrated that the pion decay source and the muon damped source can be distinguished at the $3\,\sigma$ level provided the accuracy of measurements on $R \equiv \phi(\nu_\mu)/\left(\phi(\nu_e)+\phi(\nu_\tau)\right)$ and $S \equiv \phi(\nu_e)/\phi(\nu_\tau)$ reach 10%.This paper describes simulation results on leptonic fluxes produced in ice detector and Earth-skimming detector from the same neutrino source.**

Lepton fluxes from astrophysical neutrinos interacting inside the Earth

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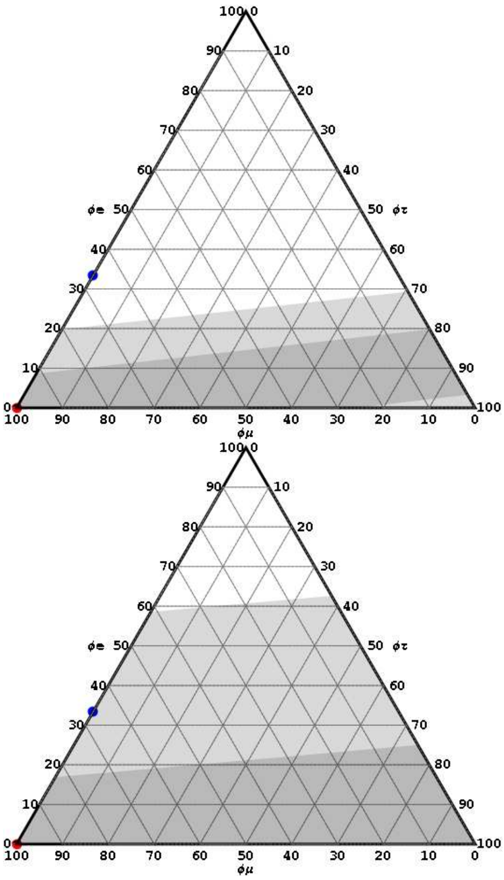

Fig. 1. Ternary plots of the neutrino source uncertainty with different precision of R and S. The left pannel is the source uncertainty with ΔR/R=10% and ΔS/S=12.57%. The right pannel is the source uncertainty only incluing ΔR/R=10%. The red point marks the standard muon source $\varphi_\mu = \{0, 1, 0\}$, the blue one is pion-damped source $\varphi_p = \{1/3, 2/3, 0\}$. Gray and light gray zone are 1σ and 3σ uncertainty range of muon source with $sin^2\theta_{13} < 0^{+0.019}$ $sin^2\theta_{23} = 0.45^{+0.09}_{-0.06}$ and CP phase therm $\delta = 0$.

I. Introduction

The operation of km^3 neutrino telescope such as IceCube [?] is an important progress in the development of neutrino astronomy [?]. But,below PeV energy in Icecube detector, tau and electron behave like showers, while muon leaves a track-like signal. It is difficult to measure the $S \equiv \phi(\nu_e)/\phi(\nu_\tau)$ under the rare neutrino events. But only measure the R ratio is ont enough to determine the neutrino flavor ratio of origion source(Fig.1). By combining Icecube with Earth-skimming neutrino detectors,which measure tau lepton exclusively, it is possible to determine exact ratio of three flavors in the Earth. Furthermore, the radio expansion of IceCube detector, the IceRay [?], is expected to detect a score of cosmogenic neutrinos per year. Such an event rate make possible to study the production mechanism of high energy neutrinos in the near future.

Recently, much effort has been devoted to studying neutrino mixing parameters with astrophysical neutrinos as the beam source [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?]. Due to the large neutrino propagation distance, the neutrino oscillation probabilities only depend on the mixing angles θ_{ij} and the CP phase δ [?], [?], which make the astrophysical beam source favorable for extracting the above parameters.

Most of the astrophysical neutrinos are believed to be produced by the decay of charged pion in the following chain: $\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_\mu + \nu_e + \bar{\nu}_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \rightarrow e^- + \bar{\nu}_\mu + \bar{\nu}_e + \nu_\mu$. This leads to the neutrino flux ratio $\phi_0(\nu_e) : \phi_0(\nu_\mu) : \phi_0(\nu_\tau) = 1 : 2 : 0$ at the astrophysical source. Such a flux ratio results from an implicit assumption that muons decay before losing significant amount of energies. In some sources, muons interact with strong magnetic fields or matter [?], [?] before they decay. The lower energy of muons give rise to ν_e with energies much lower than those of ν_μ. Consequently such a source has a neutrino flavor ratio $\phi_0(\nu_e) : \phi_0(\nu_\mu) : \phi_0(\nu_\tau) = 0 : 1 : 0$, which is referred to as the damped muon source. The third type of source emits neutrons resulting from photo-disassociation of nuclei. As neutrons propagate to the Earth, $\bar{\nu}_e$ are produced from neutron β decays [?], leading to a neutrino flavor ratio $\phi_0(\nu_e) : \phi_0(\nu_\mu) : \phi_0(\nu_\tau) = 1 : 0 : 0$. Finally, neutrinos might be produced deep inside optically thick sources so that the flavor ratio at the source surface is significantly different from the flavor ratio at the production point due to the matter effect [?]. In such a case, the fraction of tau neutrinos ϕ_{ν_τ} can be significant, unlike the previous three cases.

As mentioned before, almost all previous studies take astrophysical neutrinos as beam sources for extracting neutrino mixing angles [?]. To have a better determination of mixing angles, for instance θ_{23}, a combined analysis of terrestrially measured neutrino flavor ratios from different astrophysical sources, such as the pion decay source and the damped muon source, has been considered [?]. A legitimate question to ask is whether it is possible to distinguish the pion decay source from the damped muon one or not. The answer of this question depends on our knowledge of neutrino mixing angles and the accuracies one can achieve in the measurement of neutrino flavor ratio such as $R \equiv \phi(\nu_\mu)/\left(\phi(\nu_e)+\phi(\nu_\tau)\right)$ and $S \equiv \phi(\nu_e)/\phi(\nu_\tau)$. In this article, we shall answer this question with a statistical analysis.

We begin with a brief review on current understanding of neutrino mixing angles in Sec. II. The exact analytic form of probability matrix that links the initial neutrino flavor ratio to the ratio measured on Earth is also given in the same section. In Sec. III, we present our results on the reconstruction of neutrino flavor ratio at the source from the data generated with chosen true values of flavor ratio and best-fit values of neutrino mixing parameters. The statistical analysis is performed with different measurement accuracies on R and S, as well as different ranges of neutrino mixing angles.

II. THEORETICAL ANALYSIS

A. Neutrino mixing angles and oscillations of astrophysical neutrinos

The neutrino flux at the astrophysical site and that detected on the Earth is related by

$$\begin{pmatrix}\phi(\nu_e)\\ \phi(\nu_\mu)\\ \phi(\nu_\tau)\end{pmatrix} = \begin{pmatrix}P_{ee} & P_{e\mu} & P_{e\tau}\\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau}\\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau}\end{pmatrix}\begin{pmatrix}\phi_0(\nu_e)\\ \phi_0(\nu_\mu)\\ \phi_0(\nu_\tau)\end{pmatrix} = P\begin{pmatrix}\phi_0(\nu_e)\\ \phi_0(\nu_\mu)\\ \phi_0(\nu_\tau)\end{pmatrix} \quad (1)$$

where $\phi(\nu_\alpha)$ is the neutrino flux measured on the Earth while $\phi_0(\nu_\alpha)$ is the neutrino flux at the source, and the matrix element $P_{\alpha\beta}$ is the probability of the oscillation $\nu_\beta \to \nu_\alpha$. The exact analytic expressions for the components of P are given by

$$\begin{aligned}
P_{ee} &= \left(1-\frac{1}{2}\omega\right)(1-D^2)^2 + D^4,\\
P_{e\mu} &= \frac{1}{4}(1-D^2)\left[\omega(1+\Delta)+(4-\omega)(1-\Delta)D^2\right]\\
&+ \frac{1}{4}(1-D^2)\left[2\sqrt{\omega(1-\omega)(1-\Delta^2)}D\cos\delta\right],\\
P_{e\tau} &= \frac{1}{4}(1-D^2)\left[\omega(1-\Delta)+(4-\omega)(1+\Delta)D^2\right]\\
&- \frac{1}{4}(1-D^2)\left[2\sqrt{\omega(1-\omega)(1-\Delta^2)}D\cos\delta\right],\\
P_{\mu\mu} &= \frac{1}{2}\left[(1+\Delta^2)-(1-\Delta)^2D^2(1-D^2)\right]\\
&- \frac{1}{8}\omega\left[(1+\Delta)^2+(1-\Delta)^2D^4\right]\\
&- \frac{1}{8}\omega\left[(1-\Delta^2)D^2(2+4\cos^2\delta)\right]\\
&- \frac{1}{2}\sqrt{\omega(1-\omega)(1-\Delta^2)}\left[(1+\Delta)-(1-\Delta)D^2\right]D\cos\delta,\\
P_{\mu\tau} &= \frac{1}{2}(1-\Delta^2)(1-D^2+D^4)\\
&- \frac{1}{8}\omega\left[(1-\Delta^2)(1+4D^2\cos^2\delta+D^4)-2(1+\Delta^2)D^2\right]\\
&+ \frac{1}{2}\sqrt{\omega(1-\omega)(1-\Delta^2)}\Delta(1+D^2)D\cos\delta,\\
P_{\tau\tau} &= \frac{1}{2}\left[(1+\Delta^2)-(1+\Delta)^2D^2(1-D^2)\right]\\
&- \frac{1}{8}\omega\left[(1-\Delta)^2+(1+\Delta)^2D^4-(1-\Delta^2)D^2(2+4\cos^2\delta)\right]\\
&+ \frac{1}{2}\sqrt{\omega(1-\omega)(1-\Delta^2)}\left[(1-\Delta)-(1+\Delta)D^2\right]D\cos\delta, \quad (2)
\end{aligned}$$

where $\omega \equiv \sin^2 2\theta_{12}$, $\Delta \equiv \cos 2\theta_{23}$, $D \equiv \sin\theta_{13}$ and δ the CP phase. In the limit $\Delta = 0 = D$, i.e., $\theta_{23} = \pi/4$ and $\theta_{13} = 0$, $P_{e\mu} = P_{e\tau}$ and $P_{\mu\mu} = P_{\mu\tau} = P_{\tau\tau}$. In this case, the probability matrix P is singular. In general, this singularity is only slightly broken since one expects Δ and D are both small.

B. Statistical Analysis

To determine the neutrino flavor ratio at the source with a statistical analysis, we employ the following best-fit values and 1σ ranges of mixing parameters [?]

$$\sin^2\theta_{12} = 0.32^{+0.02}_{-0.02}, \sin^2\theta_{23} = 0.45^{+0.09}_{-0.06}, \sin^2\theta_{13} < 0.019. \quad (3)$$

We also consider the possibility of a non-zero θ_{13} suggested in Ref. [?] where

$$\sin^2\theta_{13} = 0.016 \pm 0.010(1\,\sigma) \quad (4)$$

from a global analysis.

In this work, pion and damped muon sources are chosen cases in which we investigate the uncertainties in reconstructing the flavor ratio at the source by neutrino-telescope measurements and the knowledge of neutrino mixing angles. The impact of the CP phase δ is also studied. Then we try to extract required statistics to distinguish between the two sources.

For the measurement by future neutrino telescopes, we consider a 10% error on both R_π and R_μ. Assuming a Poisson distribution of the number of events per neutrino flavor leads to $\Delta S/S = \sqrt{S(1+S^{-1})^2/(1+R^{-1})}(\Delta R/R)$. Using central values from Eq. (??), we obtain $\Delta S_\pi/S_\pi = 1.2(\Delta R_\pi/R_\pi)$ and $\Delta S_\mu/S_\mu = 1.3(\Delta R_\mu/R_\mu)$.

With these quantities, we construct

$$\chi^2 = \sum_Y \left[\frac{<Y> - Y(R,S,\theta_{ij},\delta)}{\sigma_Y}\right]^2, \quad (5)$$

where $<Y>$ is the central value of the theoretical description $Y(R,S,\theta_{ij},\delta)$ for the observable Y.

Our statistical analysis is performed by handling the quantity χ^2 with respect to the two dimensional subset (R,S) out of the parameter space (R,S,θ_{ij},δ). For a certain source with a specified flavor ratio, the parameters are scanned over the 1σ and 3σ ranges with respect to the corresponding central values. The neutrino flavor

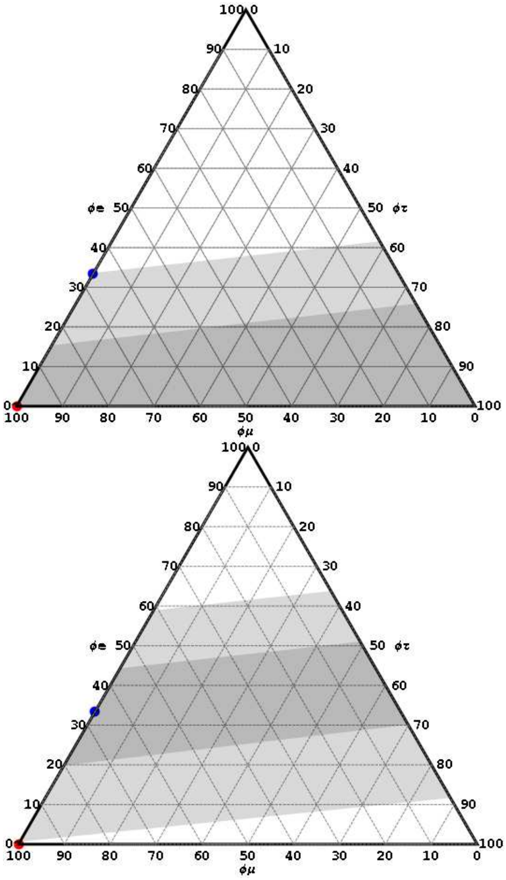

Fig. 2. the source uncertainty with different $\triangle$R. Gray and light gray denote the 1σ and 3σ boundaries of source uncertainty. The 3σ boundary of muon source uncertainty decouples from pion source, $\varphi_p = \{1/3, 2/3, 0\}$, with $\triangle$R/R=18.25% (left pannel). Pion source decouples when $\triangle$R/R=12.75% (right pannel). The mixing angle of plots set as $sin^2\theta_{13} < 0^{+0.019}$, $sin^2\theta_{23} = 0.45^{+0.09}_{-0.06}$

ratio at the source is accordingly determined to 1σ and 3σ range for $\chi^2 < 2.3$ and $\chi^2 < 11.8$.

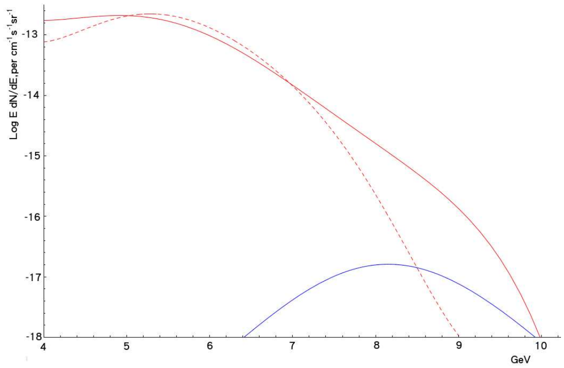

Fig. 3. The Nwutrino source fluxes model in SHINIE.the red line is MPR agn neutrino Flux [?],dashed one is SDSS agn neutrino flux [?] [?]. The Blue line is GZK neutrino flux [?]. [?]

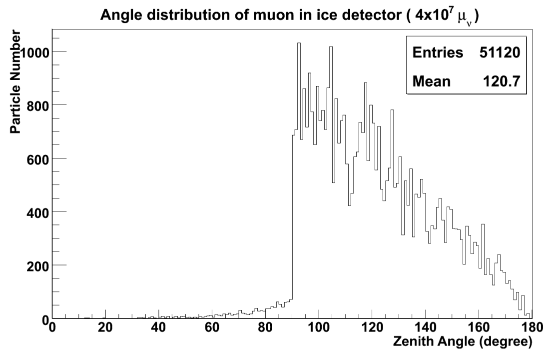

Fig. 4. The zenth angle distribution of muon in the ice detector. It generated by $10^{15} - 10^{18}$eV agn muon neutrino flux interact with Earth.

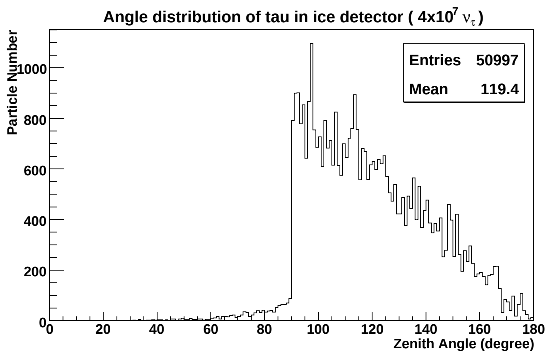

Fig. 5. The zenth angle distribution of muon in the ice detector. It generated by $10^{15} - 10^{18}$eV agn tau neutrino flux interact with Earth.

III. Lepton Fluxes Simulation

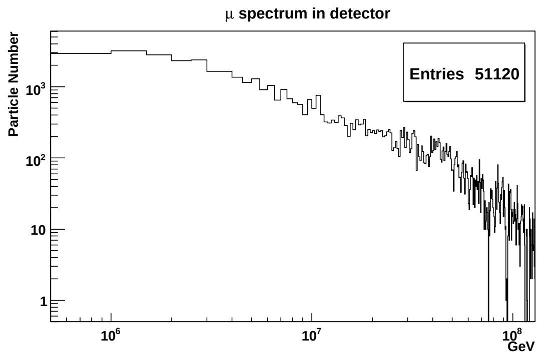

Fig. 6. The muon spectrum in the ice detector. The $10^{15}-10^{18}$eV agn neutrino flux interact with Earth and generate the muon in the ice detector.

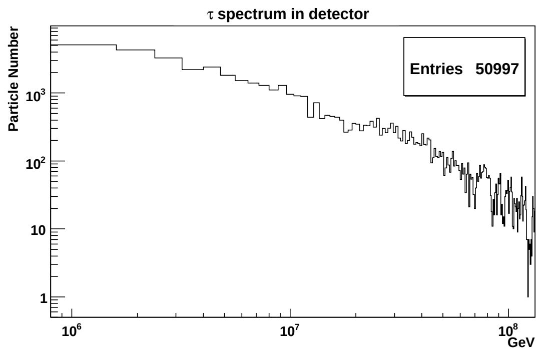

Fig. 7. The tau spectrum in the ice detector.The $10^{15}-10^{18}$eV agn neutrino flux interact with Earth and generate the tau in the ice detector.

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