Modelling the compact pulsar wind nebula of the Vela supernova remnant

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***Abstract*. Observations of the $3-10$ keV X-ray emission from the compact Vela pulsar wind nebula show a steepening of the photon index with an increase in distance from the pulsar. Using a time-independent model, this steepening effect could be satisfactorily reproduced using a single power-law lepton injection spectrum. A magnetic field strength of $B_{\rm s} = 62\ \mu$G was found at the shock, with the ratio of electromagnetic to total particle energy ranging from $\sigma = 0.01 - 0.10$, depending on the value used for the compression ratio at the shock. More importantly, a conversion efficiency of pulsar spin-down power to lepton luminosity of $\eta = 0.13$ was found. This is considerably larger than the value of $\eta = 0.003$ found by integrating over the entire lifetime of the pulsar and accompanying *H.E.S.S.* observations. This apparent discrepancy implies that a significant amount of leptons has escaped from the pulsar wind nebula. If the reverse shock caused a radial component in the pulsar wind magnetic field (it is usually assumed that pulsar wind nebulae have an azimuthal field), particles diffusing along the field lines would have a larger possibility of escape, in contrast to the diffusion of particles across the magnetic field. Using the same model to fit the radio data was unsuccessful, indicating that a second leptonic component must be present in the pulsar wind nebula.**

***Keywords*: pulsar wind nebula, X-ray, Vela**

I. Introduction

The Vela supernova remnant (SNR), located at a distance of ≈ 290 pc [4], is one of the few known SNR's consisting out of both a shell component and a pulsar wind nebula (PWN). One distinct feature of the Vela PWN, commonly referred to as Vela X, is that it has an elongated shape, with the pulsar located towards the northern edge of the PWN (see e.g. [1]). This is in contrast to other PWN'e, where the PWN is located (relatively) symmetric around the pulsar. The asymmetry of the Vela PWN is believed to be due to the shell component expanding into a region of non-homogeneous density. Magnetohydrodynamic (MHD) simulations have shown that a reverse shock will form in the denser region, with the reverse shock propagating back towards the central pulsar, thereby displacing any PWN present [2].

A common property of all PWN'e is that the non-thermal emission is visible on a large wavelength scale. Observations of the non-thermal emission from Vela X include the Inverse Compton component observed by *H.E.S.S.* [1], as well as X-ray synchrotron data observed by *XMM-Newton* in the $3-10$ keV energy range [16]. The X-ray data is of particular interest due to the fact that it was possible to obtain photon indices at a number of distances from the pulsar, ranging from $0.'5 - 15'.0$ ($0.042 - 1.3$ pc). The X-ray data clearly showed the photon index changing from $\Gamma = 1.5$ to $\Gamma = 2.0$ as a function of distance from the pulsar. This steepening in the photon index can directly be related to the change in the lepton spectrum responsible for the non-thermal emission. In turn, the change in the lepton spectrum can be explained by the adiabatic and synchrotron losses that the leptons experience while propagating away from the shock of the PWN.

This paper presents the results found from modelling the X-ray synchrotron emission of Vela X in the vicinity of the pulsar. This is the first detailed quantitative investigation into the magnetisation of the Vela PWN shock. Any attempt to model synchrotron emission from a PWN often reduces to solving a single problem: finding the correct initial lepton injection spectrum at the PWN shock, as well as its evolution as it is transported away from the shock. Since the same lepton spectrum is responsible for both the synchrotron and Inverse Compton emission, it is thus also possible to "predict" the very high energy gamma-ray emission of the compact nebula, as seen by *H.E.S.S.*

II. The Model

A. The lepton injection spectrum and its evolution

Particles undergoing shock acceleration will have a power-law energy spectrum, but taking into account the radio observations of PWN'e, it has been suggested that the lepton injection spectrum at the shock of a PWN should actually be given by a broken power-law of the form [19]

$$Q(E_{\rm e}) = \begin{cases} Q_0(E_{\rm e}/E_{\rm b})^{-\alpha_1} & \text{if } E_{\rm e} < E_{\rm b} \\ Q_0(E_{\rm e}/E_{\rm b})^{-\alpha_2} & \text{if } E_{\rm e} \geq E_{\rm b}, \end{cases}$$

where Q_0 is a normalisation constant, $E_{\rm e}$ the lepton energy, and $E_{\rm b}$ an unknown break energy. The low energy component is responsible for the radio emission, whilst the high energy component is responsible for at least X-ray emission. Since the modelling process focuses on the X-ray emission, the lepton injection spectrum can be reduced to a single power-law. It is also important to note that the model is time-independent. This is sufficient when only modelling the X-ray emission close to the pulsar where freshly injected leptons are expected.

The constant Q_0 can be determined by requiring that

$$\int_{E_{\rm b}}^{E_{\rm max}} Q(E_{\rm e})E_{\rm e}{\rm d}E_{\rm e} = \eta L. \quad (1)$$

The factor η is the conversion efficiency of spin-down power, L, to lepton luminosity at the shock. The position of the shock was found to be at $0'.35$ [17], which is comparatively close to the first measurement made by [16] at $0'.5$. If the leptons have undergone negligible energy loss, a photon index of $\Gamma = 1.5$ translates into a lepton power-law index of $\alpha = 2$. Taking the second part of the conditional equation, Equation (1), and $\alpha = 2$, one obtains

$$Q_0 = \frac{\eta L}{E_{\rm b}^2}\frac{1}{\ln(E_{\rm max}/E_{\rm b})}. \quad (2)$$

The expression for the lepton injection spectrum thus reduces to the simple form

$$Q(E_{\rm e}) = \frac{\eta L E_{\rm e}^{-2}}{\ln(E_{\rm max}/E_{\rm b})}. \quad (3)$$

It can seen from Equation (3) that the lepton injection spectrum is largely independent of $E_{\rm b}$. The only dependence is hidden in a logarithmic factor which is relatively insensitive to any variation in the value of $E_{\rm b}$.

The simplest way to solve the evolution of the lepton spectrum is to determine the time any given lepton takes to travel between two points, and then to calculate the corresponding energy loss. Because synchrotron losses are proportional to $E_{\rm e}^2$, the high energy leptons will lose their energy the fastest, thus converting the lepton spectrum from a single into a broken power-law. With an increase in distance from the pulsar, the break continually moves to lower energies.

B. Constraints on the model

While performing the modelling process, a spherical symmetry was assumed. Although Vela X is by no means spherical, the region under consideration is close enough to the pulsar to be considered as spherical. Let r denote the distance from the pulsar, while v is the speed of the leptons (taken to be uniform for all energies), and B the magnetic field at position r. It is also assumed that $\mathbf{v} \sim v\hat{\mathbf{e}}_{\rm r}$ and $\mathbf{B} \sim B\hat{\mathbf{e}}_{\phi}$.

The first, very important, constraint on the model comes from describing the conservation of magnetic flux in a steady-state MHD approximation [12] as

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = 0. \quad (4)$$

If it is assumed that the radial flow velocity $\mathbf{v}$ can be expressed as a function of r, the obvious choice that presents itself as an expression for the velocity is

$$\mathbf{v} = v_{\rm s}\left(\frac{r_{\rm s}}{r}\right)^{\delta}\hat{\mathbf{e}}_{\rm r}, \quad (5)$$

where δ is a free parameter specifying the radial profile of the velocity, while the subscript s denotes values at the shock. Thus, by combining Equations (4) and (5), one can obtain the radial profile of the magnetic field B, provided that an expression or value for B_s is available. Such an expression was derived by [13], and given as

$$B_{\rm s} = \kappa\left[\frac{\sigma L}{(1+\sigma)c}\right]^{1/2}\frac{1}{r_{\rm s}}, \quad (6)$$

where κ is the compression ratio at the shock, and σ the ratio of electromagnetic to total particle energy at the shock. In order to simplify the modelling process, κ and σ were combined into a single parameter

$$\Lambda = \kappa\sqrt{\frac{\sigma}{1+\sigma}}. \quad (7)$$

For relativistic shocks, the value for κ can vary from $< 1 - 3$.

A second important constraint concerns the maximum energy $E_{\rm max}$ of the electrons in the PWN. In the case of a weak magnetic field, the constraint states that the Larmor radius, $r_{\rm L}$, of an accelerated particle cannot be larger than the shock radius ($r_{\rm s}$) of the PWN. This restriction on the movement of a particle ensures that the particle will remain confined within the shock whilst still participating in the acceleration process. This restriction can be expressed as [7]

$$E_{\rm max} = e\epsilon B_{\rm s} r_{\rm s}, \quad (8)$$

with e denoting electron charge, and $\epsilon \leq 0.5$ being a containment factor (for more details, see [7]). Taking $\epsilon = 0.5$, $B_{\rm s} = 50\ \mu$G, and $R_{\rm s} = 9.2 \times 10^{16}$ [17], one finds that $E_{\rm max} \approx 1$ PeV.

Finally, a 'constraint' can also be placed on the break energy, $E_{\rm b}$, of the lepton injection spectrum. For a lepton of a given energy $E_{\rm b}$ (erg), in a magnetic field of strength B (gauss), the break in the synchrotron spectrum, $\nu_{\rm b}$, is given by [15]

$$\nu_{\rm b} \approx 6.3 \times 10^{18} B E_{\rm b}^2\ {\rm Hz}. \quad (9)$$

For Vela X, the break frequency should lie somewhere between $\nu_{\rm b} \approx 2.4 \times 10^{10}$ Hz (radio observations) and $\nu_{\rm b} \approx 2.4 \times 10^{17}$ Hz (X-ray observations). Assuming $B = 5\ \mu$G, this implies that 20 GeV $< E_{\rm b} <$ 55 TeV.

Furthermore, due to the nature of Equation (3), varying the value of $E_{\rm b}$ should not drastically affect the modelling results. If one takes $E_{\rm max} \approx 1$ PeV, and one varies $E_{\rm b}$ between $m_{\rm e}c^2$ and 100 GeV, this only causes a difference of about a factor 2 in the value of $\ln(E_{\rm max}/E_{\rm b})$.

III. RESULTS AND DISCUSSION

Figure 1 is a comparison between the results obtained from the model and the experimental results of [16]. The left panel is a comparison of the photon index, and the right panel a comparison of the synchrotron flux. It can be seen that the model is in relative good agreement with the experimental results, with a combined Chi-squared

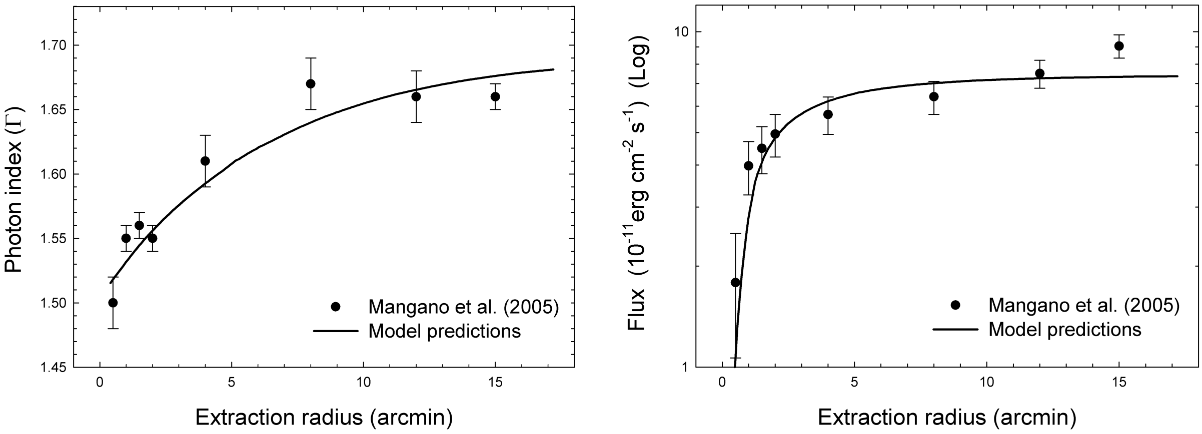

Fig. 1: Comparison between the modelling results and the results of [16]. *Left*: photon index, and *right*: the synchrotron flux

TABLE I: Best-fit values for the free parameters

Parameter	Description	Value
η	conversion efficiency of spin-down power to lepton energy	0.13
δ	radial profile parameter	0.55
ϵ	containment factor	0.33
Λ	combined parameter	0.32
$B_{\rm s}$ (μG)	magnetic field strength at shock	62
σ	ratio of electromagnetic to total particle energy at shock	$0.01 - 0.1$ [1]

value of 0.95. Table I lists the best-fit values for the free parameters, while also giving the possible range of σ values. All values were calculated for $E_{\rm b} = 100$ GeV.

Putting the best-fit parameters into Equation (6) results in $B_{\rm s} = 62$ μG, dropping to $B = 8$ μG at a distance of $r = 15'.0$, and $B = 4$ μG at $r = 1°$. This is in good agreement with the average value of $B = 5$ μG found by [14] and [8]. However, the best-fit value for $\eta = 0.13$ is considerably larger that the value of $\eta = 0.003$ found by the latter authors from *H.E.S.S.* and X-ray observations. In this paper we only consider the present day instantaneous pulsar input as probed by the compact nebular observations, whereas [7] integrated the spin-down power over the entire lifetime of the pulsar, and compared this total energy with the total energy in Vela X, as derived from *H.E.S.S.* observations. This discrepancy in the value of η between the model and [8] leads to the conclusion that a significant amount of leptons had to escape from the pulsar wind nebula ([11] referred to a "missing" leptonic component). This also implies that if no leptons had escaped, the very high energy gamma-ray signal, as seen by *H.E.S.S.*, would have been considerably larger, and brighter.

The loss of leptons can possibly be attributed to the effects of the reverse shock. With the assumption that $\mathbf{v} \sim v\hat{\mathbf{e}}_{\rm r}$ and $\mathbf{B} \sim B\hat{\mathbf{e}}_{\phi}$, any diffusion that takes place will have to be cross-field diffusion. This cross-field diffusion is, however, relatively ineffective, thus enabling PWN'e to contain particles efficiently [7]. It was also noted that if diffusion parallel to the magnetic field were to take place (if the magnetic field had a radial component), the PWN's ability to confine particles would be lessened. MHD simulations (Ferreira, personal communication) have shown that an effect of the reverse shock is to create a radial component in the magnetic field that stretches from the pulsar to the tip of the PWN. It thus follows that particles would be able to escape from the PWN along the radial component of the magnetic field following the passage of the reverse shock.

The amount of leptons lost can be reduced by increasing the value of $E_{\rm b}$. Choosing $E_{\rm b} = 15$ TeV results in $\eta = 0.05$ and a reduced Chi-squared value of 1.5. $E_{\rm b} = 15$ also represents the largest possible break energy. Any further increase in $E_{\rm b}$ makes it impossible to find an acceptable fit to the X-ray data.

One of the aims of this modelling process was to find the value for σ, but since the value of κ is unknown, σ can only be restricted to the interval shown in Table I. This interval agrees well with what other authors have found. [3] surmised that $0.01 \leq \sigma < 0.1$, while [18] found that $0.05 \leq \sigma < 0.5$. The latter authors found that $\kappa = 2.3$ and $\sigma = 0.1$ gave a best-fit for their model. Inserting $\kappa = 2.3$ into the current model result in $\sigma = 0.02$.

Figure 2 shows the synchrotron spectra obtained from the model for radial distances of $1'.5$, $4'.0$ and $12'.0$. Also shown are the corresponding *XMM* spectra, as well as the $INTEGRAL$ spectrum. The scatter plots are radio and *OSSE* data. It can be seen that the model fits

[1] κ ranges from $1.01 - 3$, with the lower σ value corresponding to $\kappa = 3$.

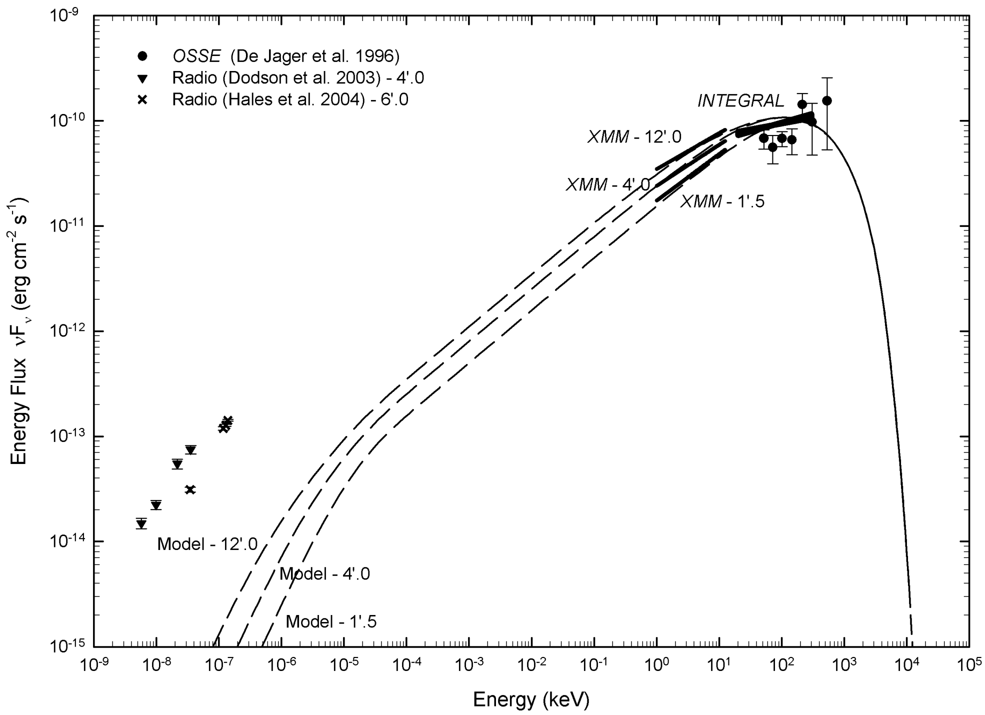

Fig. 2: Synchrotron spectra obtained with the model are shown for radii of $1'.5$, $4'.0$, and $12'.0$. Experimental spectra include the corresponding *XMM* spectra, as well as the *INTEGRAL* spectrum. Scatter plots are the *OSSE* data, and two sets of radio data.

the XMM spectra quite well, but has some trouble in fitting the $INTEGRAL$ spectrum. Also noticeable is that the extrapolated X-ray spectra are below the radio data, indicating that a second lepton component must be introduced if one wants to model the total synchrotron emission from Vela X. This same conclusion was also discussed by [8].

IV. Conclusions

Comparing the value of η found from the model with the value derived from the integrated spin-down power over the entire lifetime of the pulsar suggests that a significant amount of leptons have escaped from the PWN. This can be attributed to the radial component created in the magnetic field by the reverse shock. This radial component allows diffusion along the magnetic field, thus enabling the leptons to escape.

The wind magnetisation parameter could unfortunately only be restricted to the interval $0.01 \leq \sigma \leq 0.1$ due to the uncertainty of the value for κ. The interval is, however, in good agreement with what other authors found.

In order to account for the radio data, a second leptonic component must be present. The maximum energy ($E_{\rm b}$) of this second lepton spectrum is currently unknown due to a lack of experimental data. Radio data, together with results from the model, suggest that it must lie between 10 GeV and 15 TeV.

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