On the exact 2D transport equation for the GCR intensity averaged over the heliolongitude

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***Abstract*. Extending our previous efforts we try to construct the exact 2D transport equation for the galactic cosmic ray intensity averaged over the longitude and the 3D equation for the longitudinal variation of the intensity. The procedure to solve them is considered. Then the zero approximation for the averaged intensity is considered for different models of the current sheet drift velocity and in the same approximation the variation of the intensity is estimated and discussed.**

***Keywords*: 2D and 3D transport equations, intensity averaged over the longitude**

I. Introduction

Usually and especially in the epochs of low solar activity the longitudinal variation of the GCR intensity in the inner heliosphere is small, i. e., its variation is mainly two-dimensional (2D; r, ϑ). However, it is believed that the proper modeling of the longitudinal effects on the GCR intensity is possible only by solving the 3D (r, ϑ, φ) equation, [8]. Nevertheless, the major part of the GCR modeling in the inner heliosphere is fulfilled using the 2D models, simulating the heliosperic current sheet (HCS) effects in different ways, [10],[3],[1].

Here we further develop our approach, [4],[5],[6], to get and solve the 2D transport equation for the GCR intensity averaged over the longitude. In Section II, starting from the full 3D equation we derive the equivalent set of two equations: 1) the 2D equation for the intensity averaged over the longitude (or over the period of solar rotation) and 2) the 3D equation for the variation of the intensity, the difference between the actual intensity and the averaged one. The advantages of this procedure are discussed. In Section III we postulate different importance of different terms in the equation for the variation of the intensity and derive the set of the equations for the approximation of both the average intensity and its variation. The zero approximation for the averaged intensity is and compared for different models of the heliospheric current sheet (HCS) effects. The advantages and shortcomings of different approaches are discussed. Then the longitudinal variation of the intensity in zero approximation is calculated, illustrated and discussed.

II. Main equations

Note that below, discussing different equations, we always call the function to be found the "intensity", $\mathcal{J}(\mathbf{r}, p, t)$, although actually the equations are for the omnidirectional distribution function, $\mathcal{U}(\mathbf{r}, p, t) = \mathcal{J}(\mathbf{r}, p, t)/p^2$. The distribution of the cosmic ray intensity in the heliosphere is usually described by the well-known equation

$$\frac{\partial \mathcal{U}}{\partial t} - \nabla(\mathcal{K}\nabla\mathcal{U}) + \left(\vec{\mathcal{V}}^{sw} + \vec{\mathcal{V}}^{d}\right)\nabla\mathcal{U} - \frac{\mathcal{D}\mathcal{V}^{sw}}{3}p\frac{\partial \mathcal{U}}{\partial p} = 0 \qquad (1)$$

where $\mathcal{K}$, $\vec{\mathcal{V}}^{d}$, $\vec{\mathcal{V}}^{sw}$ and $\mathcal{D}\mathcal{V}^{sw}$ are the diffusion tensor, particle drift and solar wind velocities and the latter's divergence, respectively.

First we derive the 2D equation for the intensity averaged over the longitude, $U = \mathcal{U} - u$, which is just equal to $\sim$ 27-day mean intensity for steady U. The coefficients of the equation can be decomposed in a similar way ($\vec{\mathcal{V}}^{sw} = \mathbf{V}^{sw} + \mathbf{v}^{sw}, \vec{\mathcal{V}}^{d} = \mathbf{V}^{d} + \mathbf{v}^{d}, \mathcal{K} = K + k, \mathcal{D}\mathcal{V}^{sw} = DV^{sw} + dv^{sw}$). However, instead of formulating the equations for U and u using the averaged coefficients and their variations, let us here for short rewrite the eq. (1) in abbreviated form as

$$\frac{\partial \mathcal{U}}{\partial t} + \mathcal{G}\cdot\mathcal{U} = 0, \qquad (2)$$

where $\mathcal{G} \equiv -\nabla(\mathcal{K}\nabla\cdot) + \left(\vec{\mathcal{V}}^{sw} + \vec{\mathcal{V}}^{d}\right)\nabla - \mathcal{D}\mathcal{V}^{sw}/3p\partial/\partial p$ is the operator acting on $\mathcal{U}$ and characterized by the set of the coefficients $\mathcal{K}, \vec{\mathcal{V}}^{sw}, \vec{\mathcal{V}}^{d}, \mathcal{D}\mathcal{V}^{sw}$, so that the operators corresponding to the averaged coefficients and their variations can be denoted as $G \equiv -\nabla(K\nabla\cdot) + \left(\mathbf{V}^{sw} + \mathbf{V}^{d}\right)\nabla\cdot + ...$ and $g \equiv -\nabla(k\nabla\cdot) + \left(\mathbf{v}^{sw} + \mathbf{v}^{d}\right)\nabla\cdot + ...$ Averaging (2) over the longitude one can get

$$\frac{\partial U}{\partial t} + G\cdot U = -\langle g\cdot u\rangle_{\varphi} \qquad (3)$$

where $\langle ...\rangle_{\varphi}$ means the averaging over the longitude. Then by subtracting (3) from (2) the following equation for u can be obtained

$$\frac{\partial u}{\partial t} + \{g\cdot u + G\cdot u - \langle g\cdot u\rangle_{\varphi}\} = -g\cdot U \qquad (4)$$

The set (3-4) with the boundary and initial conditions (not to be discussed here) is equivalent to (2). If one neglects the right-hand side in (3), this equation will describe the cosmic ray propagation in the longitudinally averaged heliosphere. This axisymmetric heliosphere may have strange features, e.g., non-divergence-free magnetic field, as was noted in [8]. It is just the source in the right-hand side of (3), describing the contribution to the average density from the asymmetrical parts of the modulation factors and intensity, which makes the 2D equation for U the exact one.

So if one knows u and puts it in rhs of (3), this equation can be used not only for simulating the main effects of the HCS in the GCR intensity, but to model these effects in full measure. Of course, to find u one should solve the 3D equation (4). In [4] - [6] we considered different approaches to estimate u without solving the full (4). We shall try to apply one of them in the next section.

The GCR intensity changes with time not only due to the rotation of the Sun (as, e. g., HMF's polariry is fixed in the rotating frame), $\partial/\partial t = -\omega\partial/\partial\varphi$, where ω is the angular velocity of the Sun. In general the GCR intensity changes also due to the variations in the averaged over the longitude characteristics, modulating the intensity. So it is advantageous to derive the 2D equation for the GCR intensity averaged over the period of solar rotation T (that is, over the time during T). Besides, it is averaging over T, $\langle ...\rangle_T$, what we can easily get from the observations, not $\langle ...\rangle_\varphi$. Naturally, eqs. (3 - 4) don't change if we mean by the $U = \mathcal{U} - u$ and $G = \mathcal{G} - g$ the characteristics averaged over the period of solar rotation.

III. On zero approximations for U and u

As in [4], we suggest that all terms containing u and its space and momentum derivatives collected in the figure brackets in the left-hand side of (4) are small when compared with its right-hand side. In short it means that in case of small u, $u \ll U$, it arises as the perturbation of the averaged intensity by the φ-varying factors of the heliosphere and its own redistribution can be neglected.

Without loss in generality we consider the most simple (and widely used) heliospheric model where the solar wind is radial with constant velocity and the only φ-dependent characteristic is the polarity of the regular magnetic field. Then the only term in the lhs of eq. (1) which needs averaging is that with $\vec{\mathcal{V}}^d$. Then in the stationary case ($\partial U/\partial t = 0$, $\partial u/\partial t = -\omega\partial u/\partial\varphi$) we have for the i-approximation of U^i and u^i:

$$-\nabla(K\nabla U^i) + (\mathbf{V}^{sw} + \mathbf{V}^d)\nabla U^i - \frac{DV^{sw}}{3} p \frac{\partial U^i}{\partial p} = -\langle \mathbf{v}^d \nabla u^{i-1}\rangle_\varphi \quad (5)$$

$$u^i = \frac{1}{\omega}\int_0^\varphi \mathbf{v}^d \nabla U^i d\varphi' + u^i(\varphi = 0) \quad (6)$$

For 0-approximation the rhs of (5) is zero. The second term in the rhs of (6) makes $\langle u^i\rangle_\varphi = 0$ fulfilled.

A. The magnetic drift in 0-approximation

For the simplest case we consider the heliospheric magnetic field (HMF) represented as $\vec{\mathcal{B}} = \mathcal{F}\vec{\mathcal{B}}^m$, where $\vec{\mathcal{B}}^m$ is the unipolar (or "monopolar") magnetic field equal to $\vec{\mathcal{B}}$ in the positive sectors and having the reversed polarity in the negative ones. The heliospheric magnetic field polarity $\mathcal{F}$ is a scalar function equal to $+1$ in the positive and -1 in negative sectors, changing from one to another on the CS surface $\mathcal{F}(r, \vartheta, \varphi, t) = 0$.

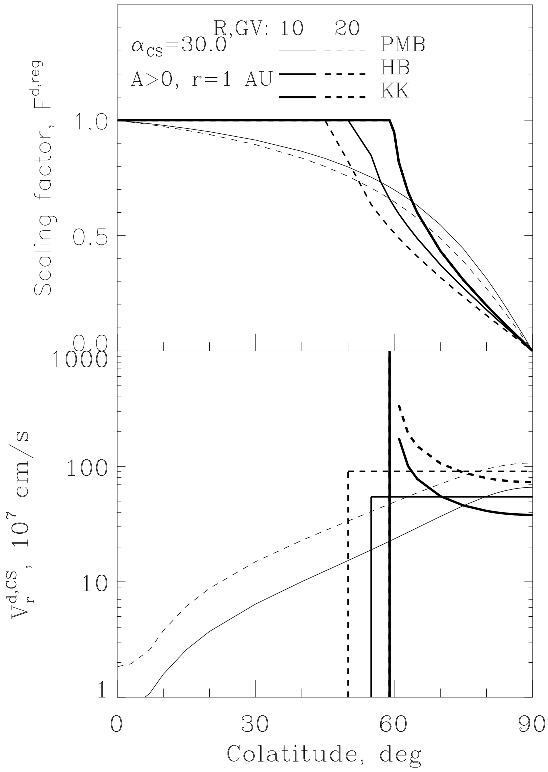

Fig. 1: The comparison of the scaling factor of the regular magnetic drift (upper panel) and the current sheet drift velocity (lower panel) according to different models.

The 3D particle drift velocity is $\vec{\mathcal{V}}^d = pv/3q\left[\nabla \times (\vec{\mathcal{B}}/\mathcal{B}^2)\right]$, [11], where v and q are the particle speed and charge, respectively. In our case one can decompose the drift velocity into the regular $\vec{\mathcal{V}}^{d,reg}$ and current sheet $\vec{\mathcal{V}}^{d,cs}$ ones, $\vec{\mathcal{V}}^{d,reg} = pv/3q\mathcal{F}\left[\nabla \times (\vec{\mathcal{B}}^m/\mathcal{B}^2)\right]$, $\vec{\mathcal{V}}^{d,cs} = pv/3q\left[\nabla\mathcal{F} \times (\vec{\mathcal{B}}^m/\mathcal{B}^2)\right]$.

In general $\mathcal{F} = A\left[1 - 2H(x)\right]$, with A and $H(x)$ being the polarity of the HMF in the high-latitude N-hemisphere and the step function of the distance x from the sheet, respectively. The value of x is positive in the positive "magnetic hemisphere". In this case $\nabla\mathcal{F} = 2\delta(x)\mathbf{n}^{cs}$,where $\mathbf{n}^{cs}$ is the unit vector normal to HCS surface and directed to the positive hemisphere. Then the averaged over the longitude drift velocities are as follows

$$\mathbf{V}^{d,reg} = \frac{pv}{3q} F \cdot \left[\nabla \times \frac{\mathbf{B}^m}{B^2}\right] \quad (7)$$

$$\mathbf{V}^{d,cs} = \frac{1}{\pi}\frac{pv}{3q}\frac{1}{Br\sin\vartheta} \cdot \sum_{i=1}^{i=2k} \left.\frac{\mathbf{n}^{cs} \times \mathbf{b}^m}{|\mathbf{n}^{cs} \cdot \mathbf{e}_\varphi|}\right|_{\varphi_i}, \quad (8)$$

where $F = \mathcal{F} - f$ is averaged over the longitude HMF polarity, $\mathbf{b}^m$ is the unit vector along the "monopole"

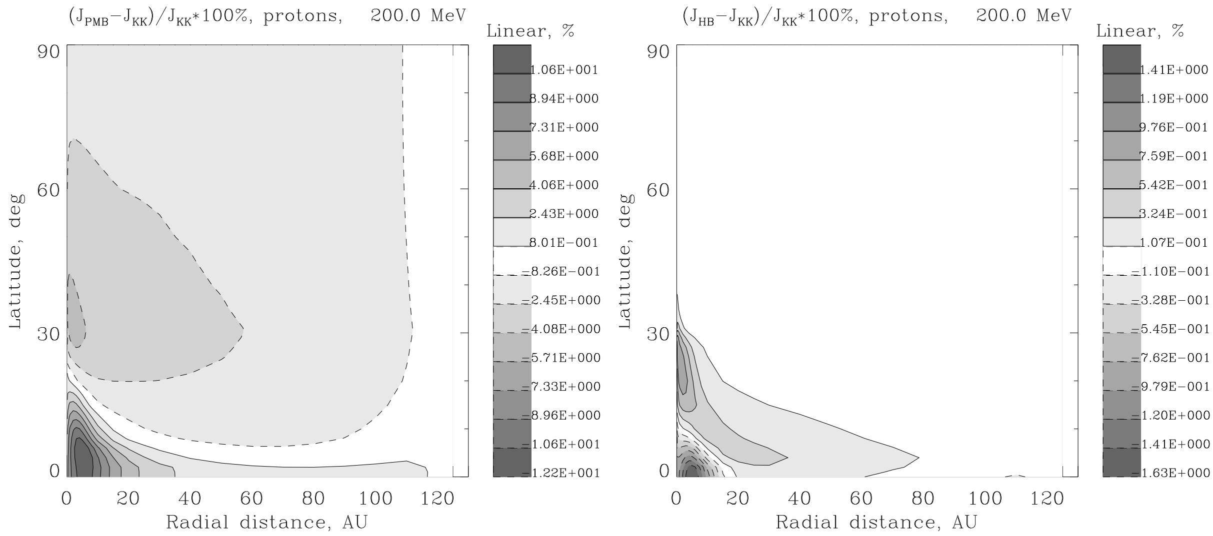

Fig. 2: The relative difference between the GCR intensity for protons with $T_n = 200MeV$ as function of r, ϑ with HCS-effects according to PMB and KK models (left panel) and according to HB and KK models (right panel) .

magnetic field line and the summation is performed for all points ($\varphi_i, i = 1, \cdots, 2k$) of intersection of the circle $r, \vartheta = const$ with the HCS surface. These expressions are valid for any HCS surface.

Then the scaling factor for the regular drift velocity and the averaged over the longitude current sheet drift velocity look as follows

$$F = \frac{2A}{\pi} \arcsin(\cot\alpha^{cs}\cot\vartheta) \quad (9)$$

$$V_r^{d,cs} = \frac{2A}{\pi}\frac{pv}{3q}\frac{\sin\chi}{Br\sin^2\vartheta}\frac{\cot\alpha^{cs}}{\sqrt{1-\cot^2\alpha^{cs}\cot^2\vartheta}} \quad (10)$$

$$V_\vartheta^{d,cs} = 0, \quad (11)$$

if ($\pi/2-\alpha^{cs} < \vartheta < \pi/2+\alpha^{cs}$), and $\mathcal{F} = \pm 1, \mathbf{V}^{d,cs} = 0$ outside this HMF sector-structure zone. Note that although the expression (10) for $V^{d,cs}$ diverges at $\vartheta_0 = \pi/2 \pm \alpha^{cs}$, it makes it very slowly ($V^{d,cs} \propto |\vartheta - \vartheta_0|^{-\frac{1}{2}}$), so there is no jump in the latitudinal gradient of the intensity at this colatitude.

In Fig. 1 the colatitude profiles of the scaling factor of the regular magnetic drift (upper panel) and the current sheet drift velocity (lower panel), derived in this work (KK-model), are compared with those obtained in [10], [2] (PMB-model) and [3] (HB-model), see [7]. We noted the main differences between these models: 1) for our model the HCS effects in F and $V^{d,cs}$ exist only in the sector-structure zone, while those in PMB- and HB-models extend to higher latitudes, the higher latitude for the greater rigidity of the particles; 2) the latitude dependence of the current sheet drift velocity is different for three models considered: for our model $V^{d,cs}$ is greatest at the maximum extend of HCS, ϑ_0; it is greatest at the equator for PMB-model; and it does not depend on latitude for HB-model. Note that the results of the numerical model [1] (AUMK-model) are similar to ours in this respect, at least partially..

We are sure that, because of the finite giroradius of the particles, there is a physical sense in the widening with rigidity of the latitude range where the particles feel the HCS, so our model should be modified in this respect. On the other hand the flat current sheet velocity field supposedly used in HB-model is valid only for homogeneous HMF (or particles with low rigidity) and formally it could not be used for high rigidity particles. The AUMK-model uses the complicated procedure to cope with this difficulty. Besides, in order to ensure that the resultant drift velocity is divergence-free, the current sheet drift velocity in HB-model was averaged over the latitude within the effective HCS latitude range. However, it could be misleading, if one is interested in the details of the GCR distribution in this latitude range.

Our opinion is that the main cause of the shortcomings of all models for the HCS drift velocity is the use of the oversimplified model of the current sheet. So we believe that the proper way to construct the consistent drift velocity field is to use more realistic model of the heliospheric current sheet (see the discussion in [9]).

To study the influence of the HCS-model on the GCR intensity we solved (5) without the rhs for U^0 with $\mathbf{V}^d cs$ according to PMB-, HB- and KK- models. The other coefficients of (5), their dependence on r, ϑ, p, the "initial" and boundary conditions etc are the simplest and widely used. As in [9] we found useful to use the 3D presentation, $\delta J(r, \vartheta)/J$, to illustrate how the different WCS models influence the intensity and to decide how they can be checked.

In Fig. 2 the dependence of the GCR intensity on the HCS-models used is shown for the drift models

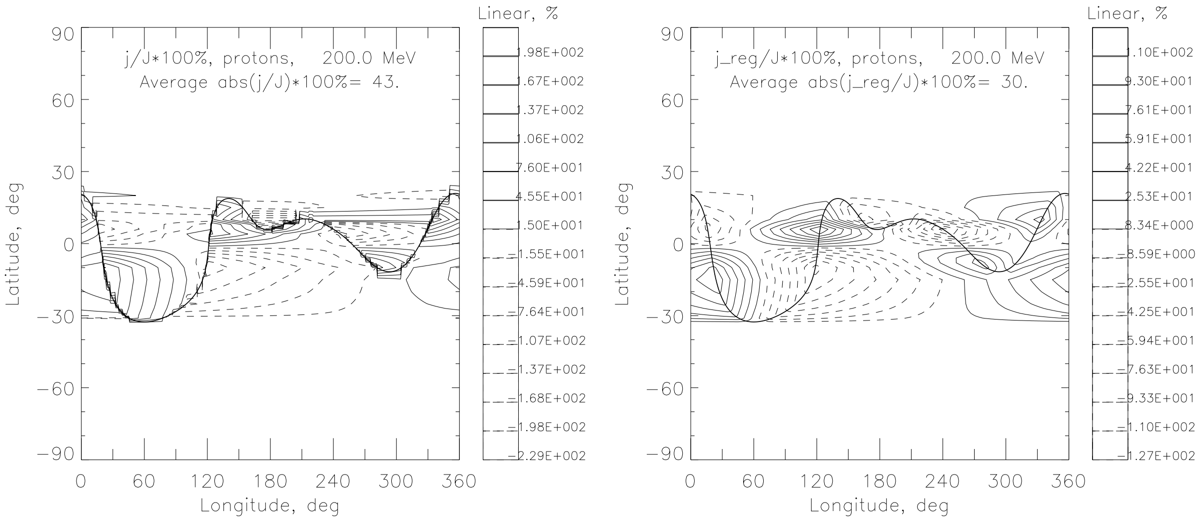

Fig. 3: The ratio of the longitudinal variation of the intensity to the averaged intensity according to (12) for CR1858 and for protons with $T_n = 200MeV$ at $r = 1AU$ as function of ϑ, φ. In the left panel the effect of both regular and current sheet drift is shown while the right panel shows only effect of the regular drift.

considered and the medium energy protons. One can see that for this ($A > 0$)-period this dependence is greatest in the inner heliosphere ($r < 10$ AU) at low and middle latitudes ($\vartheta > 50°$). For the higher energy the value of this difference is smaller. So for $A > 0$-periods the different WCS-models can be checked better using the medium energy nuclei aboard near-the-Earth and Ulysses spacecraft.

B. The longitudinal variation of the intensity in 0-approximation

In 0-approximation the expression for the longitudinal variation of the intensity is as follows (omitting the 0 subindex):

$$u = \int_0^{\varphi} (\mathcal{F} - F) d\varphi' \cdot \frac{1}{\omega} \mathbf{V}^{d,m} \nabla U + \frac{pv}{3qB} \cdot \left[\int_0^{\varphi} (\nabla \mathcal{F} - \langle \nabla \mathcal{F} \rangle_{\varphi}) d\varphi' \times \mathbf{b}^m \right] \quad (12)$$

As in [9] we showed that it was preferable to use the real form of HCS instead the model ones, in Fig. 3 the relative to the averaged intensity longitudinal variation of the intensity to the averaged intensity according to (12) for CR1858 ($A > 0$) and for protons with $T_n = 200MeV$ at $r = 1AU$ as function of ϑ, φ. It can be seen that the effect of the whole current sheet drift on variation of the longitudinal variation of the intensity is rather complicated and difficult to understand. The effect of the regular drift is much more understandable and can be formulated as growth (decrease) of the intensity near the antiHale- (Hale-) sector boundaries. For opposite polarity of the heliospheric magnetic field the effects will be opposite.

However, the first thing one notices is that the maximum longitudinal variation of the intensity is even higher than the averaged intensity, i. e., the condition used for getting the expression (6) for u^i, is violated. But the relative longitudinal variation of the intensity averaged over the sector zone (shown in the figure) is much smaller. Besides, the use of i-th approximation for u to get the higher approximation for U etc can lead in the limit to smaller u.

Given the 0-approximation to the longitudinal variation of the intensity, one can calculate its gradient and then find the rhs of the equation for the 1-approximation to the averaged intensity. We shall discuss it later.

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