Fluctuation of (Ultra-)High Energy Muons and the Influence of their Production Spectrum over their Definite Path Length

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***Abstract*. As far as the solutions of the high energy muon propagation in the medium by the Monte Carlo methods are concerned, there are two approaches to this problem, namely, [differential method] and [integral method]. The majority concerned is occupied by [differential method] group among which results by GEANT 4 are included, while we belong to the minority group in which [integral method] is developed.**

In our older paper ([integral method]) which was published in 18-th ICRC, Bangalore, India[1], we developed a new technique around the range fluctuation of high energy muon, taking into account of all the stochastic processes in the fluctuation as exactly as possible by the Monte Carlo method.

In our present paper, we change all cross sections concerned in our older paper to the most new ones which are utilized in GEANT 4 and we study the range fluctuation of (ultra-)high energy muons in detail by the exact treatment on the stochastic processes of (ultra-)high energy muons.

In the poster presentation, direct comparison between the results obtained by [differential method] and those obtained [integral method] are made.

***Keywords*: Muon Propagation, Fluctuation, Neutrino**

I. Reliability of the Monte Carlo method concerned.

Cosmic rays phenomena essentially belong to rare events, in which, therefore, fluctuations may decisively important role. The Monte Carlo method is an indispensable means for the analysis of cosmic ray events. However, at the same time, this method should be recognized rather risky one, because, generally speaking, to prove the validity of the Monte Carlo method concerned is very difficult.

As far as the programs for the muon propagations by the Monte Carlo method are concerned, there are now four programs are available for the analysis of the events concerned and design studies for the possible KM3 detector, namely PROMU[2], MUSIC[3], MUM[4], MMC[5], and MUSAN[6].

The fundamental techniques utilized in these Monte Carlo programs are essentially same. They divide physical processes contributed to the muon propagation into two parts, namely, [the fluctuation part] and [continuous part] as for the treatment of their energy losses. Then, they adopt, $0.05 \sim 10^{-3}$ a numerical value of the characterized quantity, v_{cut}. Their results obtained should be coincided with each other within plausible errors and such the situation is quite understandable, because their philosophy to solve the problems is same each other. However, the validity of their philosophy adopted are not proven by methodologically different and independent means.

There are two different approaches to solve the muon propagation problem by the Monte Carlo method. One is called [differential method] and the other is called [integral method]. These four programs are made on the [differential method].

In 1983, Takahashi *et al.* [1] presented a new approach to solve high energy muon propagation problems by the Monte Carlo method and they applied the [integral method][1] to the analysis of the depth intensity of muons.

On the other hand, starting from the fundamental equation for electron shower under Approximation B in the Nishimura-Kamata formalism, Misaki and Nishimura solved the muon propagation problems under some cross sections analytically and exactly[7].

Then, Takahashi *et al.* [1] compared muon energy spectrum at different depths and the effective energies of muons at different depths by analytical method[7] with the corresponding results by their Monte Carlo method, and showed the excellent agreement the analytical solutions and theirs. As the analytical theory on the muon propagation[7] is methodological different from the Monte Carlo one, the agreement between them shows the validity of their Monte Carlo method.

Now, we change older cross sections in Takahashi *et al.* to the most new one which GEANT 4 utilizes, never changing the fundamental structure of the older Monte Carlo Method.

II. Exact consideration into fluctuation

We point out the logical inconsistency in well-utilized programs for muon propagation in the matter by using [differential method][2], [3], [4], [5], [6].

As we mention above, they introduce the concept of v_{cut} in their Monte Carlo programs in order to consider fluctuation in the muon propagation. However

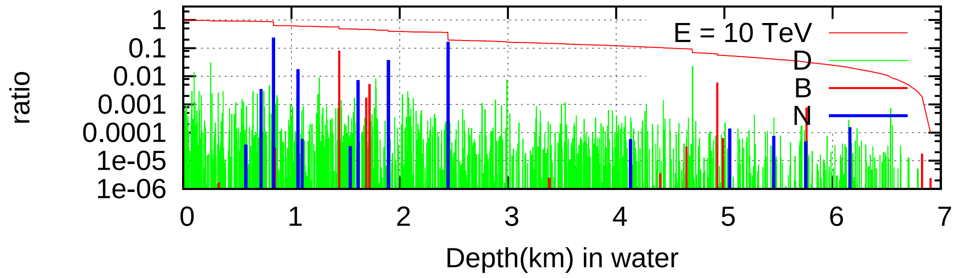

Fig. 1

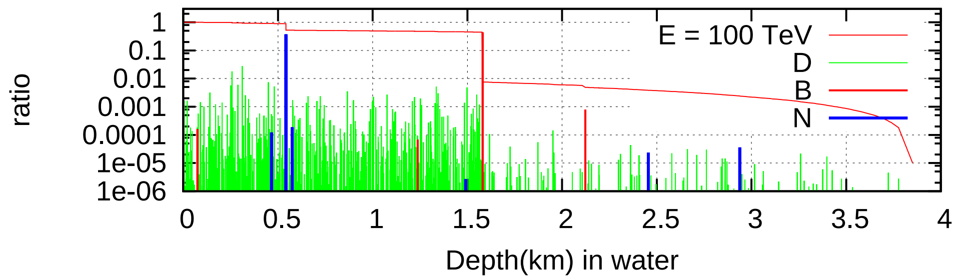

Fig. 2

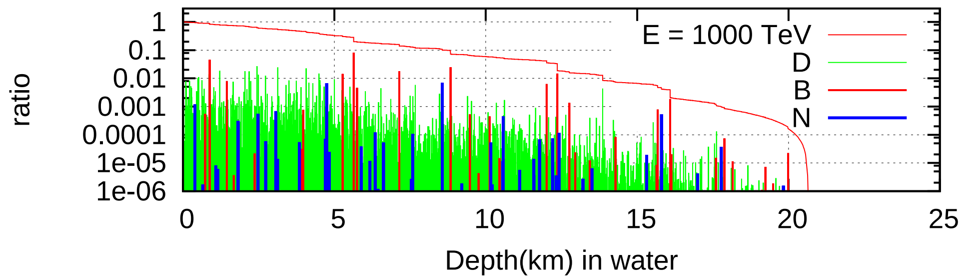

Fig. 3

they fix the numerical value of v_{cut} throughout their calculation. Now we restrict our discussion to the case by bremsstrahlung only, for the sake of convenience. In this case, when some muon with a given energy contributes to [fluctuation part], another muon with the same energy may contributes to [continuous part] and, consequently, the muons with same energies are treated in different manners without unified selection rule in their Monte Carlo method. Furthermore, the introduction of such definite value into v_{cut} deforms the energy distribution of the radiated gamma rays unnaturally. It should be noticed that the almost Cherenkov radiations from (ultra)-high energy muons are generated by the accompanied gamma rays around the muons concerned, not by the muons themselves (the dressed muons) [8] and, therefore, such the deformation of gamma rays distribution around the muons may mislead the energy estimation on the muons concerned by Cherenkov radiations.

On the other hand, in our integral method, v_{cut} is always taken such a way that $v_{cut} \ll E_{min}/E_0$, where E_0 is the primary energy of the muon and E_{min} is the minimum energy for observation. Such the v_{cut} adopted by our integral method guarantees the reproduction of the gamma rays distribution by bremsstrahlung exactly in our Monte Carlo method.

Now, we show our procedure based on the [integral method].

We think the fluctuation in the muon propagation comes from bremsstrahlung, direct electron pair production and nuclear interaction. Then, the mean free paths of the bremsstrahlung, electron pair production and nuclear interaction are given as follows,

$$\lambda_{b,d,n}\left(E_{\mu}\right) = \frac{1}{\int_{E_{min}}^{E_{max}} \psi_{b,d,n}\left(E_{\mu}, E\right) dE} \qquad (1)$$

,where $\psi_{b,d,n}\left(E_{\mu}, E\right) dE$ denotes the differential cross section for bremsstrahlung, direct electron pair production, and nuclear interaction, respectively. The values of E_{min} and E_{max} are adopted from kinematically allowed region.

Our Monte Carlo procedure is as follows.

procedure(1):

We first define the total mean free path in the

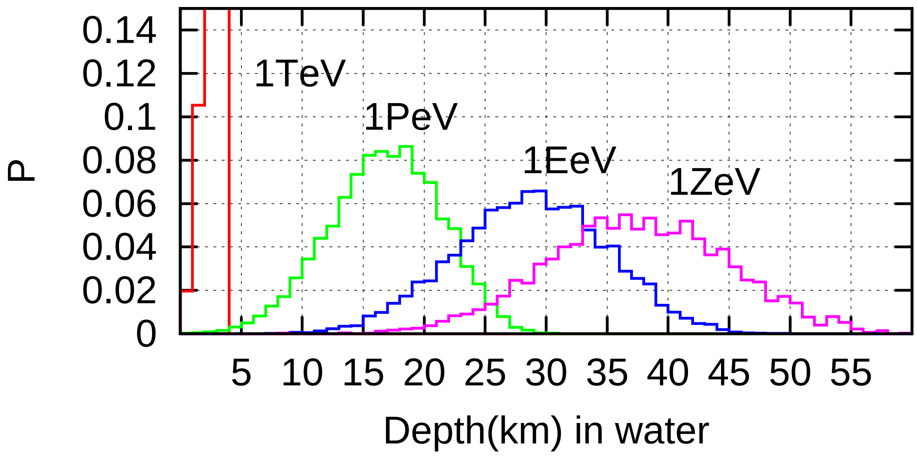

Fig. 4

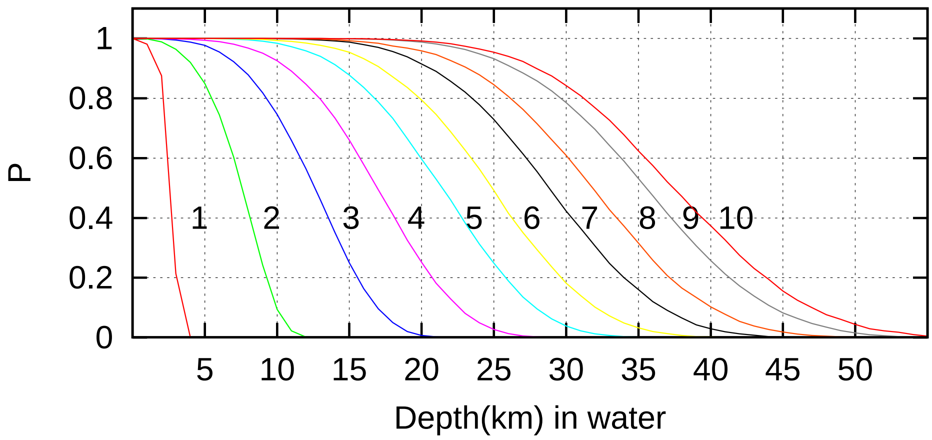

Fig. 5

following:

$$\frac{1}{\lambda(E_\mu)} = \frac{1}{\lambda_b(E_\mu)} + \frac{1}{\lambda_d(E_\mu)} + \frac{1}{\lambda_n(E_\mu)} \quad (2)$$

By using Eq.(2) and ξ, we determine the interaction point of the muon concerned by Eq.(3)

$$t = -\lambda(E_\mu)\ln\xi \quad (3)$$

,where ξ denote the uniform random number between(0,1). Hereafter, ξ denotes the uniform random number between (0,1).

procedure(2):

We define the following two numerical values,

$$\xi_a = \frac{1/\lambda_b(E_\mu)}{1/\lambda_b(E_\mu) + 1/\lambda_d(E_\mu) + 1/\lambda_n(E_\mu)} \quad (4)$$

$$\xi_b = \frac{1/\lambda_b(E_\mu) + 1/\lambda_d(E_\mu)}{1/\lambda_b(E_\mu) + 1/\lambda_d(E_\mu) + 1/\lambda_n(E_\mu)} \quad (5)$$

Now, we determine the kind of the interaction which occurs at the depth t.

We take ξ and ask where ξ belongs to in the following three cases,

1) If the case $(0 \le \xi < \xi_a)$, then, we judge the bremsstrahlung process occurs.
2) If the case $(\xi_a \le \xi < \xi_b)$, then, we judge the direct pair production occurs.
3) If the case $(\xi_b \le \xi < 1)$, then, we judge the nuclear interaction occurs.

procedure(3):

By the procedure(2), we determine where the muon concerned interacts with the medium and the kind of the interaction concerned.

Next, we calculate the dissipated energy due to the interaction concerned.

The dissipated energy, E_t is given as follows, taking ξ.

$$\xi = \frac{\int_{E_{min}}^{E_t} \psi_{b,d,n}(E_\mu, E)\, dE}{\int_{E_{min}}^{E_{max}} \psi_{b,d,n}(E_\mu, E)\, dE} \quad (6)$$

Then, the muon energy after interaction, $E_{\mu a}$

is given as,

$$E_{\mu a} = E_{\mu b} - \epsilon t \tag{7}$$

,where ϵ is the inonization loss per unit length and t denotes the distance between a interaction and subsequent interaction.

We repeat from procedure(1) to procedure(3) until the muon concerned either falls down below the minimum energy or it escapes from the region of our interest. Thus, we know where and which interaction occurs, and how much energy is dissipated due to the interaction concerned exactly.

III. Some examples of the muon propagation

In Figs.1 to 3, we give "personal history" of individual muon whose history is pursued until the muon energy fall down 1GeV. The length of the "needles" in the figures represent magnitude of the dissipated energies due to different interactions. Abscissa denote the distance from the starting point of the muon to the interaction point. Ordinate denote the ratio of dissipated energy due to the interaction concerned to energy of the primary muon. The colors with the needles present the kind of the interaction. D(green) denotes direct electron pair production, B(red) denotes bremsstrahlung and N(blue) denotes nuclear interaction. The ordinate axis represent the ratio of dissipated energy due to the interaction concerned to primary energy.

Figs 1, 2 and 3 represent the personal histories of individual muon with 10 TeV, 100 TeV and 1000 TeV, respectively. It is easily understood from Figs.1 to 3 that the energy loss due to direct electron pair production occur far frequently than those of bremsstrahlung and nuclear interaction and each energy loss due to direct pair production is rather small.

In Fig.1, we show one example of moderate energy loss by a 10 TeV muon. Rather big energy losses are caused by two nuclear interactions and one bremsstrahlung. However, it does not influence the situation globally. The most of the energy losses is due to many number of direct pair production with rather small energies. This event survive until 7 kilo meters.

In Fig.2, we show one example of the catastrophic energy losses due to nuclear interaction ($\sim$ 0.5 kilo meter from the starting point) as well as bremsstrahlung ($\sim$ 1.6 kilo meter) in the case of a 100 TeV muon. Namely, the muon concerned loses almost 99% of the primary energy by only two interactions. The muon concerned of the 100 TeV muon die at $\sim$ 4 kilo meter, which is shorter than 7 kilo meter by the 10 TeV muon in Fig.1.

In Fig.3, we show one example of a 1000 TeV muon. We can see big energy losses mainly coming from bremsstrahlung. It is easily understood from these figure that fluctuation effects in muon propagation mainly coming from bremsstrahlung and nuclear interaction and the contribution to fluctuation from direct electron pair production is rather small, although their total energy loss is big.

In Fig.4, the range distributions of the muons for 1 TeV, 1 PeV, 1 EeV and 1 ZeV are given. Each curve is normalized to unity. Their minimum energies are 1 GeV. The maximum probability of 1 TeV muon is 0.66 and it is cut down due to out of scale in Fig.4. It is easily understood from the figure that the widths of the range distribution spread, as primary energies increase. This denotes that the fluctuation effect become larger, as primary energies increase. The forward part of the range distributions is caused by bigger fluctuation due to a few number of the catastrophic energy losses coming from bremsstrahlung and nuclear interaction, while the backward part of the range distribution are caused by smaller fluctuation due to a number of direct electron pair productions with smaller energies, accompanied by the nuclear interaction and bremsstrahlung with smaller energies.

In Fig.5, we give the survival probabilities for different primary energies and the same minimum energy, 1GeV. The numerical values attached to each curve 1,2,3,4,5,6,7,8,9,10 denote the primary energies, 10^{12}eV, 10^{13}eV, 10^{14}eV, 10^{15}eV, 10^{16}eV, 10^{17}eV, 10^{18}eV, 10^{19}eV, 10^{20}eV, 10^{21}eV, respectively. It is clear from the figure that the survival probabilities strongly increase, as their primary energies increase.

IV. Conclusion

Utilizing the information mentioned above, we will give the production spectrum of muons for their given definite length in the poster session.

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