Injection spectrum of electrons in the Galactic sources of the cosmic rays

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***Abstract*. A retrieve of the injection spectrum of electrons in the Galactic sources of the cosmic rays was carried out using the observed energy spectrum of electrons in the Solar system region. Fractal diffusion model was implemented for the description of propagation of particles from sources to the Solar system.**

It was shown, that self-consistent description of experimental data could be obtained, if the source spectral index in this approach turns out to be changes from $p = 2.6$ to $p = 2.85$ at energy region $0.1 - 10^3$ GeV.

An absence of spectral feature (see, e.g., data ATIC ballon experiment) at the range $300-800$ GeV shown by the Fermi-LAT collaboration confirms our recent calculations and in a good agreement with the new calculations energy spectrum of electrons. We also have made a new calculation of the positron to electron ratio $\mathrm{e}^+/(\mathrm{e}^+ + \mathrm{e}^-)$.

***Keywords*: injection spectrum of electrons, fractal diffusion model, stable law, positron to electron ratio**

I. Introduction

Recently published experimental data on the cosmic ray electrons spectrum from PAMELA satellite on-board instrument [1], ATIC balloon-borne experiment [2], H.E.S.S. collaboration [3] and PPB-BETS flight in Antarctica [4] have opened a new phase in study of high energy electrons. PAMELA instrument found an increase of positrons with respect to electrons at energies above a few GeV. H.E.S.S. present significant steepening of the spectrum above 600 GeV. ATIC and PPB-BETS detect a spectral feature at the range 300-800 GeV. But a high precision measurement of the electron spectrum from 20 GeV to 1 TeV performed with Fermi Large Area Telescope [5] not confirm the prominent spectral feature observed by ATIC and PPB-BETS. The above mentioned results initiated the correction of the existing models of origin and propagation of cosmic ray electrons, as well as development of a new ones.

In the paper [6], for example, spectrum of electrons, observed at the experiments, was described in terms of accepted diffusive approach [7], [8], where spectrum of particle generation in a sources $S \propto E^{-2,2}$ and energy dependence of diffusivity $D \propto E^{-0,6}$, in assumption that high-energy spectrum part ($E \geq 100$ GeV) formed by local young sources ($r \leq 100$ pc, $t \leq 10^5$ y), as low-energy part formed by distant ($r \geq 1$ kpc) sources of Galaxy. In other words, in the paper [6] spectrum of electrons was reproduced by combination of the two Gaussians characterizing contributions for the two above presented groups of sources.

However, the multiple experiments of the last years show that the distribution of matter in the Galaxy is highly nonuniform and more likely of fractal type. Normal diffusion models cannot be used to describe propagation of particles in that media. Today it is well known, that an account of the fractal properties of the medium can be realize by replacement of the Laplace operator Δ in the normal diffusion equation by fractional differentiation operator on the spatial variable (see, for example, [9] and references therein, as well as the paper [10]). The fractional Laplacian [11], reflecting a presence of the large paths of the particles in a cavities, make the diffusion process of cosmic rays is non-local.

The main goals of this study are to retrieve the injection spectrum of electrons and to calculate of the positron fraction in galactic cosmic rays. The particles propagate along the source - Solar system path in a fractal-like medium according to the fractal diffusion laws.

II. Fractal diffusion model

Fractal Galaxy in the first approach could be considered as medium, which consist of magnetic clouds of different scales, situated in non-magnetic neutral gas. Particle motion represents interchange of quick “Levy flights” (linear free paths, which distributed as $p(x) \propto r^{-\alpha-1}$ where $r \to \infty$) and staying in “traps” (magnetic clouds with fractal structure). Characteristic exponent α is defined by fractal features of medium. Finite mean time of particle staying in a trap was assumed in this paper.

The equation for the concentration of electrons with an energy E, generated in a fractal medium by galactic sources with a distribution density $S(\vec{r},t,E)$, taking into account an energy losses, has the form [12]

$$\frac{\partial N}{\partial t} = -D(E,\alpha)(-\triangle)^{\alpha/2}N(\vec{r},t,E) + \frac{\partial(b(E)N(\vec{r},t,E))}{\partial E} + S(\vec{r},t,E). \quad (1)$$

Here $D(E,\alpha) = D_0(\alpha)E^{\delta}$ is the fractal diffusivity [10] and $b(E)$ is the mean rate of continuous energy loss. The fractional Laplacian $(-\triangle)^{\alpha/2}$ [9], [11] as noted above

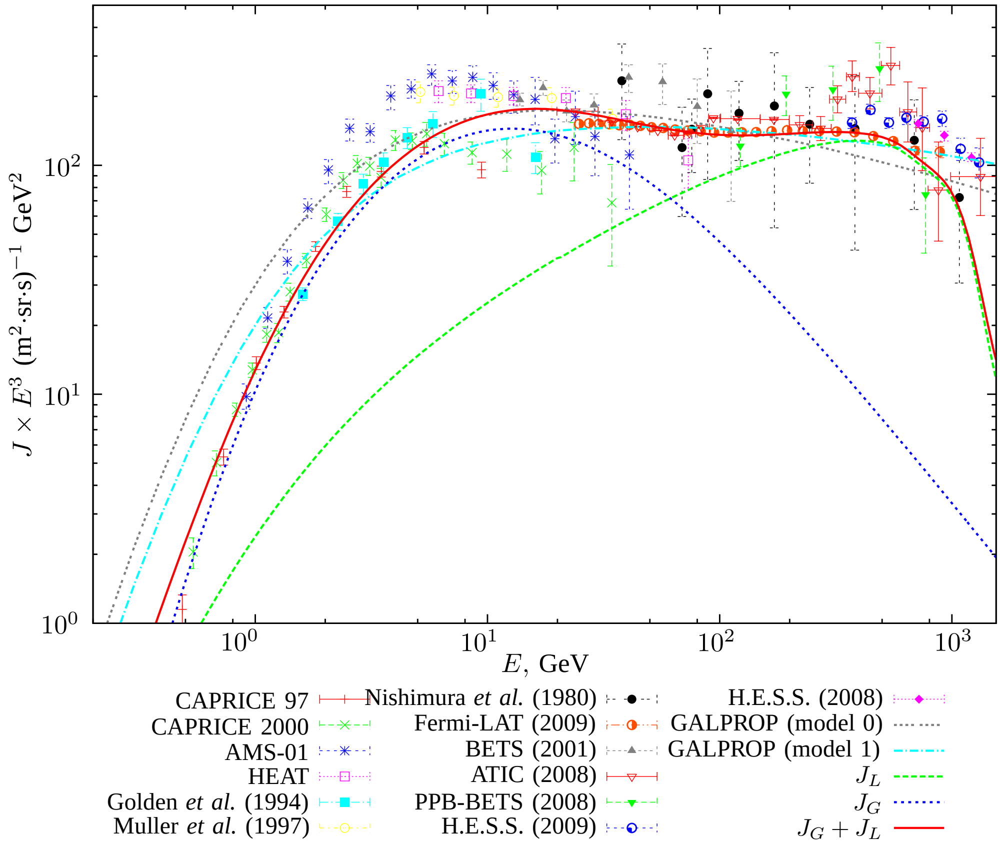

Fig. 1. Comparison of the electrons spectrum calculated within the fractal diffusion model with the experimental data presented in [12] and also with new data ATIC [2], H.E.S.S. [3], PPB-BETS [4] and Fermi-LAT [5]

reflects the presence of the large paths of electrons in the galactic fractal-like medium.

The rate of change in the energy of electrons, $b(E)$, during their propagation in the medium is attributed to ionization, inverse Compton losses, bremsstrahlung, and synchrotron radiation. According to [6], we can write $b(E)$ as

$$b(E) = b_0 + b_1 E + b_2 E^2 \approx b_2(E + E_1)(E + E_2), \quad (2)$$

where $b_0 = 3.06 \cdot 10^{-16} n$ (GeV s^{-1}), $b_1 = 10^{-15} n$ (s^{-1}), and $b_2 = 1.38 \cdot 10^{-16}$ (GeV$\cdot$ s)$^{-1}$ (for the magnetic field intensity $B = 5\ \mu$G and background photon density $\omega = 1$ (eV cm^{-3}), whereas $E_1 = b_0/b_1$ and $E_2 = b_1/b_2$.

The technique for calculating the fractal diffusivity D_0 was described in [10]. In that paper, based on the analysis of the nuclear component of cosmic rays, the exponent of the energy dependence of fractal diffusivity δ was found to be $\delta = 0.3$, that actually coincides with the value $1/3$, accepted for the Kolmogorov turbulence [13].

The equation for Green's function $G(\vec{r}, t, E; E_0)$ describing particles diffusion under condition that the electrons started from origin $\vec{r}_0 = 0$ at the time $t_0 = 0$ with the energy E_0 has the form [12]

$$\frac{\partial G}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2} G + \frac{\partial (b(E)G)}{\partial E} + \\ +\delta(\vec{r})\delta(t)\delta(E - E_0). \quad (3)$$

The Green's function of the problem was derived using standard substitutions [7] and Fourier transform:

$$G(\vec{r}, t, E; E_0) = \frac{g_3^{(\alpha)}(|\vec{r}|\lambda^{-1/\alpha})}{\lambda^{3/\alpha}(1 - b_2 t(E + E_2))^2} \times \\ \times \delta\left(E_0 - \left\{\frac{E + E_1}{1 - b_1 t(E + E_2)/(E_2 - E_1)} - E_1\right\}\right) \times \\ \times H(1 - b_2 t(E + E_2)) H(t).$$

Here, $g_3^{(\alpha)}(r)$ is the probability density of three-dimentional sphericaly-symmetrical distribution [9], [14], and

$$E_0(t) = \frac{E + E_1}{1 - b_1 t(E + E_2)/(E_2 - E_1)} - E_1,$$

$$\lambda(E, E_0) = \int_E^{E_0(t)} \frac{D(E', \alpha)}{b(E')} dE'.$$

III. Energy spectrum of electrons and positron fraction

A solution to Eq. 1 for a point impulse source with a power energy spectrum

$$S(\vec{r}, t, E) = S_i E^{-p} \delta(\vec{r}) \Theta(T - t) \Theta(t), \quad (4)$$

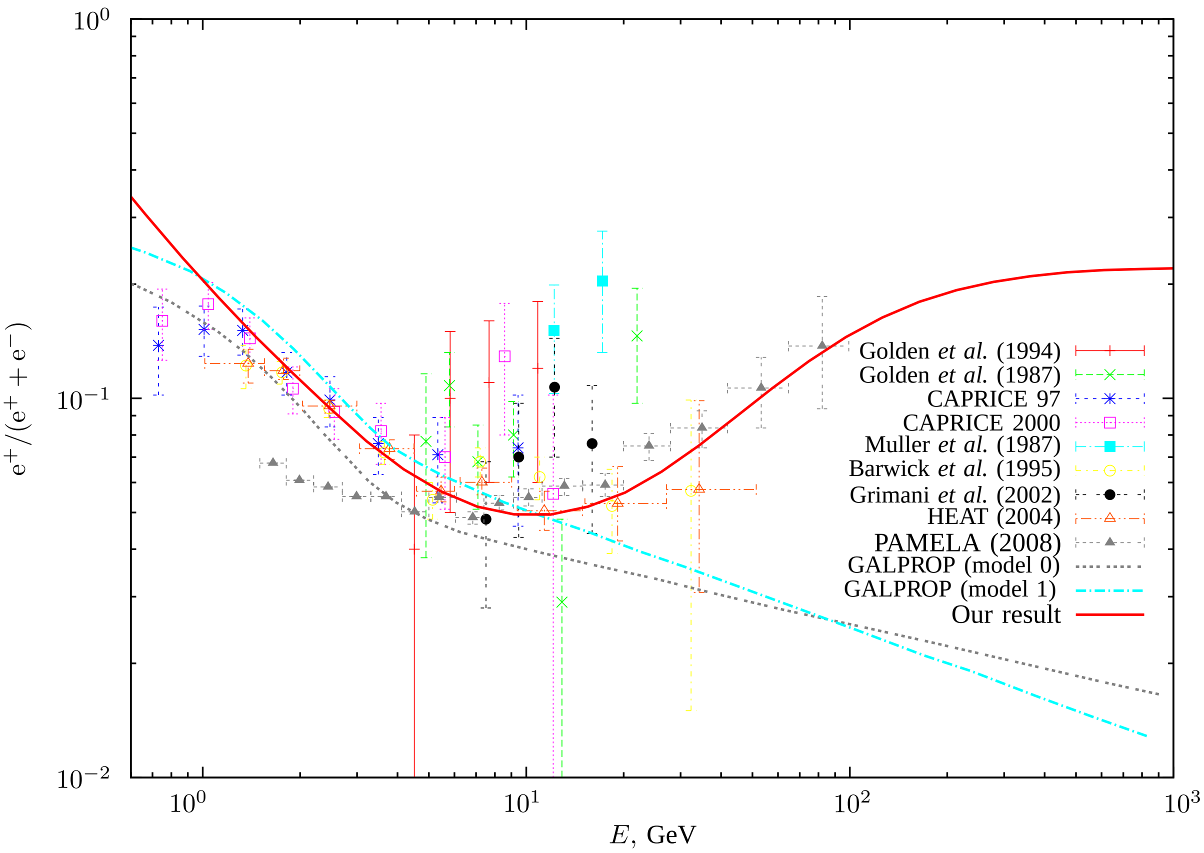

Fig. 2. Comparison of the positron fraction calculated within the fractal diffusion model with the experimental data. Results of modeling with the GALPROP code also presented in this figure

TABLE I
THE SPACE-TIME COORDINATES OF THE NEAREST SOURCES OF COSMIC RAYS ACCEPTED IN THE WORKS

r, pc	t, 10^5 y
100	2.0
175	4.0
200	4.0
210	4.0
250	0.12
300	0.86
400	3.4
400	0.38
600	0.46
600	0.32
700	0.43
700	0.32
770	0.20
800	0.23
800	0.27
900	0.47

which simulate electrons generation in supernovae, has the form

$$N(\vec{r},t,E) = S_i \int\limits_{\max[0,t-T]}^{\min[t,1/b_2(E+E_2)]} dt' E_0^{-p} \lambda(t',E)^{-3/\alpha} \times$$
$$\times (1 - b_2 t'(E+E_2))^{-2} g_3^{(\alpha)} \left(|\vec{r}| \lambda(t',E)^{-1/\alpha}\right).$$

For a point steady source

$$S(\vec{r},E) = S_c E^{-p} \delta(\vec{r}) \qquad (5)$$

the solution is

$$N(\vec{r},E) = S_c \int\limits_0^{1/b_2(E+E_2)} dt' E_0^{-p} \lambda(t',E)^{-3/\alpha} \times$$
$$\times (1 - b_2 t'(E+E_2))^{-2} g_3^{(\alpha)} \left(|\vec{r}| \lambda(t',E)^{-1/\alpha}\right).$$

According to [6], [10], the electrons intensity from all galactic sources was presented as

$$J(\vec{r},t,E) = J_G(\vec{r},E) + J_L(\vec{r},t,E) =$$
$$\frac{v}{4\pi} \left(N(\vec{r},E) + \sum_{r \le 1\text{kpc}} N(\vec{r}_j, t_j, E) \right), \qquad (6)$$

where J_G is the global spectrum component, i.e. the contribution of multiple old ($t \geq 10^6$ yr) distant ($r \geq 1$ kpc) sources and J_L is the local component determined by the nearby ($r \leq 1$ kpc) young ($t \leq 10^6$ yr) ones.

At particle energies $E \approx (0.1 \div 10)$ GeV, the electron flux observed in the Solar system is influenced by modulation effects; hence, we used the model proposed in [15], with the potential $\Phi(t) = 600$ MeV to take into account the solar modulation. In Fig. 1 we compare new Fermi-LAT cosmic ray electrons data [5], as well as several other experimental data sets (references are given in [12]) with the electron spectrum modeled within fractal diffusion model. Also in Fig. 1 we present results obtained with the GALPROP code [16] for the parameters discussed in [17].

For calculation of the positron spectrum and positron to electron ratio fractal diffusion model also was used. In our scenario we propose the Galactic sources also inject the positrons with the same source spectral index p (see (4), (5)) as in the case of the electrons. It is found that self-consistent description of positron fraction could be obtained. In Fig. 2 we present comparison of positron fraction calculated within the fractal diffusion model with the recently published results, as well as the PAMELA experiment data [1].

IV. Results and conclusion

Spectra of electrons and positrons fraction were calculated under different assumptions on the degree of galactic medium fractality (parameter α), fractal diffusivity $D_0(\alpha)$, and exponent p of the injection spectrum of cosmic ray particles in sources. Duration of particle generation in local sources was assumed to be $T \approx 10^4$ years and $\delta = 0.27$. The space-time coordinates of the nearest sources of cosmic rays accepted in the works are listed in the table I.

An analysis of the obtained results on the electron spectrum and positron fraction showed that self-consistent description of the experimental data is attained if the exponent of the injection spectrum of electrons and positrons in the Galactic sources within the energy range $0.1 - 10^3$ GeV varies from $p = 2.6$ in the low-energy range to $p = 2.85$ at $E \geq 10^3$ GeV.

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References

[1] M. Casolino, P. Picozza, F. Altamura *et al. Launch of the space experiment PAMELA* // Adv. Space Res, 2008, 42, P. 455.

[2] J. Chang, J. H. Adams Jr, H. S. Ahn *et al. An excess of cosmic ray electrons at energies of 300-800 Gev* // Nature, 2008, 456, Pp. 362-365.

[3] F. Aharonian, A. G. Akhperjanian, G. Anton *Probing the ATIC peak in the cosmic-ray electron spectrum with H.E.S.S. // ArXiV:astro-ph/0905.0105v1*.

[4] S. Torii, T. Yamagami, T. Tamura *et al. High-energy electron observations by PPB-BETS flight in Antarctica // arXiv:astro-ph/0809.0760v1*.

[5] A. A. Abdo, M. Ackermann, M. Ajello *et al. Measurement of the cosmic ray $e^+ + e^-$ spectrum from 20 GeV to 1 TeV with the Fermi Large Area Telescope // ArXiV:astro-ph/0905.0025v1*.

[6] A. M. Atoyan, F. A. Aharonjan, H. J. Völk. *Electrons and positrons in the galactic cosmic rays* // Phys. Rev. D., 1995, V. 52, Pp. 3265–3275.

[7] S. I. Syrovatskii. *Distribution of relativistic electrons in the Galaxy and spectrum of synchrotron radio emission* // Astron. Zh., 1959, 36, Pp. 17–28. (in russian)

[8] V. L. Ginzburg, S. I. Syrovatskii. *Origin of cosmic rays*, Pergamon Press, 1964.

[9] V. V. Uchaikin, V. M. Zolotarev. *Chance and stability*, VSP. Netherlands, Utrecht, 1999.

[10] A. A. Lagutin, A. G. Tyumentsev. *Spectrum, mass composition and anizotropy of cosmic rays in the fractal Galaxy* // Izv. Altai. Gos. Univ., 2004, 35(4), Pp. 4–21 (in Russian).

[11] S. G. Samko, A. A. Kilbas, O. I. Marichev. *Fractional integrals and derivatives — Theory and Applications*, Gordon and Breach, New York, 1993.

[12] A. A. Lagutin, A. G. Tyumentsev. *Spectrum of electrons in the Galaxy* // Izv. Altai. Gos. Univ., 2004, 35(4), Pp. 22–26 (in Russian).

[13] V. S. Berezinsky, S. V. Bulanov, V. L. Ginzburg *et al. Astrophysics of cosmic rays*. North Holland, Amsterdam, 1990.

[14] V. M. Zolotarev, V. V. Uchaikin, V. V. Saenko. *Superdiffusion and stable law* // ZhETF, 1999, 115, Pp. 1411–1425 (JETP (Engl. Transl.), vol. 115, p. 780).

[15] L. J. Gleeson, W. I. Axford. *Solar modulation of galactic cosmic rays* // ApJ, 1968, 154, Pp. 1011-1025.

[16] A. W. Strong, I. V. Moskalenko. *Models for galactic cosmic-ray propagation* // Adv.Space Res., 2001, 27, Pp. 717-726.

[17] D. Grasso, S. Profumo, A. W. Strong *et al. On possible interpretations of the high energy electron-positron spectrum measured by the Fermi Large Area Telescope // ArXiV:astro-ph/0905.0636v1*.