

A stochastic view of the propagation of galactic cosmic rays

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Abstract. We have estimated numerically the age and path length distributions of galactic cosmic rays (GCRs) arriving at the Solar system by a new stochastic method. In this method we utilized a coupled stochastic differential equations (SDEs) which is equivalent to the Parker's convection-diffusion equation describing the propagation of GCRs. We assume the GCRs are accelerated in supernova remnants (SNRs) and supernovae (SNe) occurred uniformly in the galactic disc and randomly in time with an average interval of 100 years during last billion years. We also assume the remnants are active for 10^5 years. We adopt for simplicity an isotropic diffusion model neglecting energy changes and the existence of the galactic wind. The trajectory of test particles is traced backward in time until the particles arrive at some active SNR. By this numerical experiment we have obtained the age and path length distributions for various population of particles with different energies. The resultant distributions are represented well by lognormal distributions.

Keywords: GCRs, age distribution, path length distribution

I. INTRODUCTION

The age and path length distributions of GCRs arriving at the solar system are the key ingredients in investigating the origin of the GCRs. Recently, a steady state semianalytical model which is based on the summation of the diffusive contributions of a myriad of spatially and temporally discrete GCR sources have been developed for calculating the age and path length distributions [1], [5]. These authors assumed GCRs are accelerated in SNRs, however, they have not taken into account fully the discreteness of occurrence of SNe both in space and time in our Galaxy. They calculated the age and path length distributions of GCRs only for the spatially and temporally averaged source distributions to compare the resulting distributions with those determined in the leaky box and standard steady state models. In this paper we propose a new and fully stochastic method to calculate the age and path length distributions of the GCRs reflecting the discreteness of occurrence of SNe. Our stochastic method depends on the well-known equivalence of a coupled SDEs to the Parker's equation describing the propagation of GCRs. The trajectory of GCRs arrived at the solar system is traced backward in time with the SDEs until the particle arrive at some

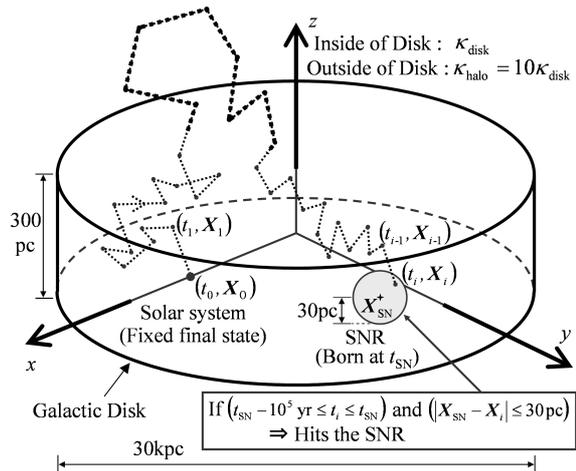


Fig. 1. Schematic view of our simulation. Particles start at the Solar system and run backward in time until they hit a SNR.

active SNR. Our new method is very versatile and is free from setting an unphysical and artificial escape boundary for GCRs required in normal investigations described in [1] and [5], for example. This technique has been applied successfully to the investigation of Solar modulation of GCRs in the heliosphere [6], [7], galactic modulation of extragalactic hypothetical CRs [3] and the structure of the CR electron halo in starburst galaxies [2]. In this paper we present the age and path length distributions and the spatial distribution of birth palaces (SNRs) of the local GCRs assuming a toy model for the structure of our Galaxy. The results for more realistic model of our Galaxy will be presented in a future paper.

II. NUMERICAL SIMULATIONS

We postulate that the GCR sources are SNRs in the Galaxy, and GCRs arrive at the Solar system after propagating in the interstellar space and/or galactic halo. We utilize a coupled SDEs which is equivalent to the Parker's convection-diffusion equation describing the propagation of GCRs,

$$d\mathbf{X} = (\nabla \cdot \boldsymbol{\kappa} - \mathbf{V}) dt + \sum_{\sigma} \alpha_{\sigma} dW_{\sigma}(t) \quad (1)$$

and

$$dp = -\frac{1}{3}p(\nabla \cdot \mathbf{V})dt, \quad (2)$$

where $\mathbf{X}=(x, y, z)$ indicates the position of particle, t is the time, \mathbf{V} is the speed of the galactic wind, p is

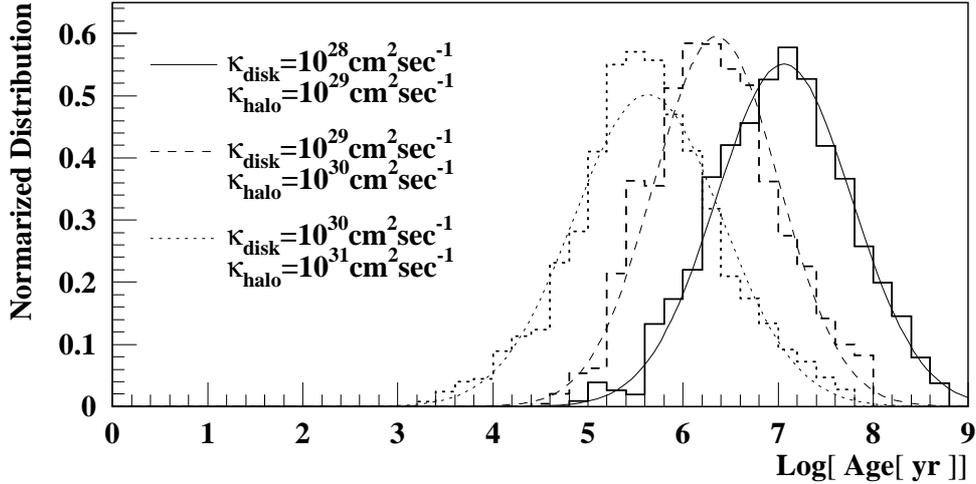


Fig. 2. CR age distributions.

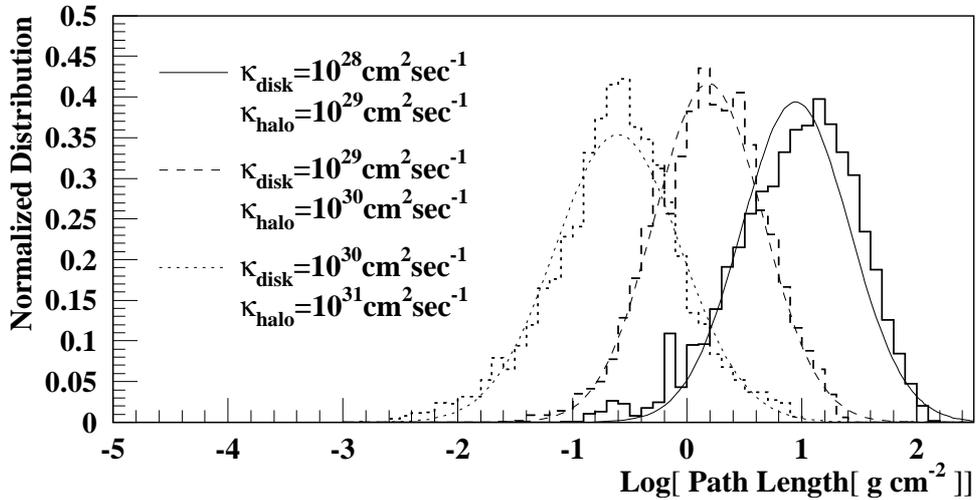


Fig. 3. CR path length distributions.

the particle momentum, κ is the diffusion coefficient tensor, $\sum_{\sigma} \alpha_{\sigma}^{\mu} \alpha_{\sigma}^{\nu} = 2\kappa^{\mu\nu}$, and dW is a Wiener process given by the Gaussian distribution $P(dW) = (2\pi dt)^{-1/2} \exp(-dW^2/2dt)$. In this study, we numerically integrate Eqs. (1) and (2) assuming for simplicity an isotropic three-dimensional diffusion model neglecting energy changes and the existence of the galactic wind; we only deal with a term $\sum_{\sigma} \alpha_{\sigma} dW_{\sigma}(t)$ in Eq. (1), since the terms $(\nabla \cdot \kappa - V) dt$ in Eq. (1) and $-\frac{1}{3}p(\nabla \cdot V)dt$ in Eq. (2) become zero in our approximation.

Fig. 1 shows a schematic view of our simulation which is represented in a galactocentric Cartesian coordinate system (x, y, z) . We assume the shape of the galactic disk is a cylinder with a radius and height of 15 kpc and 300 pc. The solar system is located at $\mathbf{X}_0 = (8.5 \text{ kpc}, 0 \text{ kpc}, 0 \text{ kpc})$. The trajectory of simulated particles is traced backward in time with a time step of Δt and a constant diffusion coefficient κ_{disk} started from the solar system at present time $t_0 = 0$ until the particles arrive at some active SNR. According to the Sedov solution of the expansion of SNRs, a radius of

SNR is estimated to be 30 pc when we assume that the SNR age, expansion energy and matter density are 10^5 yr, 10^{51} erg and 1 proton cm^{-3} , respectively. In our simulation, the active SNR is defined as the region of $|\mathbf{X} - \mathbf{X}_{SN}| \leq 30 \text{ pc}$ during $t_{SN} - 10^5 \text{ yr} \leq t \leq t_{SN}$, where \mathbf{X}_{SN} and t_{SN} are the position and time when SN occurred. Here we assume that SN occurred uniformly in the galactic disk and randomly in time with an average interval of 100 years during last billion years. The particles propagate back and forth between the galactic disk and the halo. While the particle is travelling the region outside of the disk, the diffusion coefficient is set to be $\kappa_{halo} = 10\kappa_{disk}$. We also calculate a path length for each time step Δt by $v\rho\Delta t$, where v is the speed of the particle and ρ is the matter density which is assumed separately for the inside and outside of the galactic disk as $\rho_{disk} = 1 \text{ proton cm}^{-3}$ and $\rho_{halo} = 0.1 \text{ proton cm}^{-3}$. Once the particle arrives at some active SNR, we record the position, arrival time and total path length, and then restart the same procedure again for another particle. The arrival time recorded corresponds to the age of the

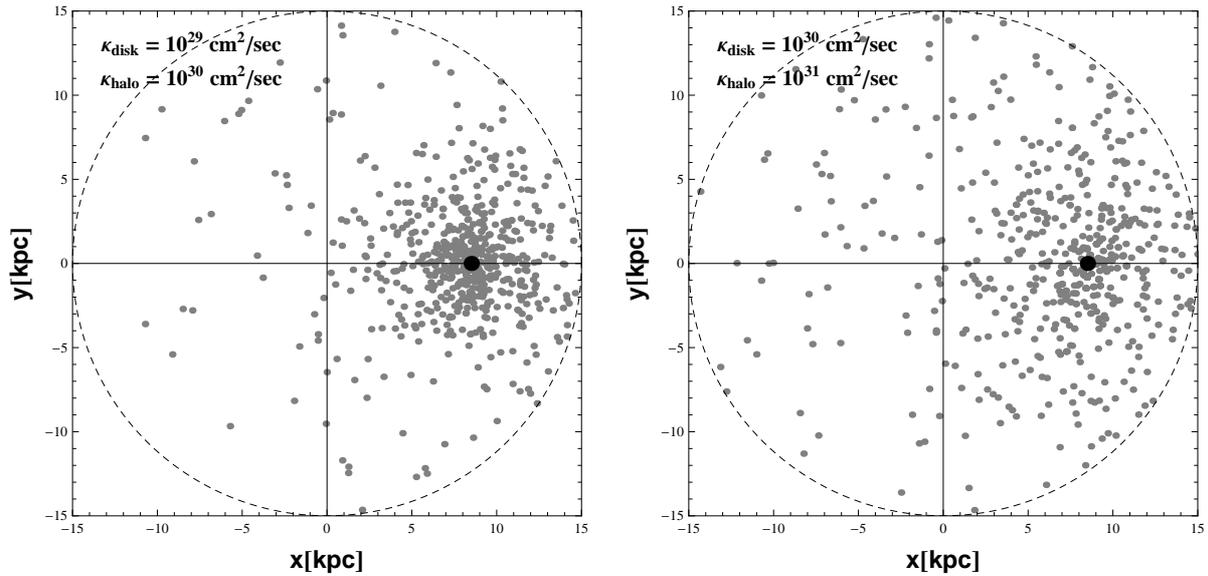


Fig. 4. CR source distributions for particles with $\kappa_{disk} = 10^{29} \text{ cm}^2 \text{ sec}^{-1}$ (left) and $\kappa_{disk} = 10^{30} \text{ cm}^2 \text{ sec}^{-1}$ (right).

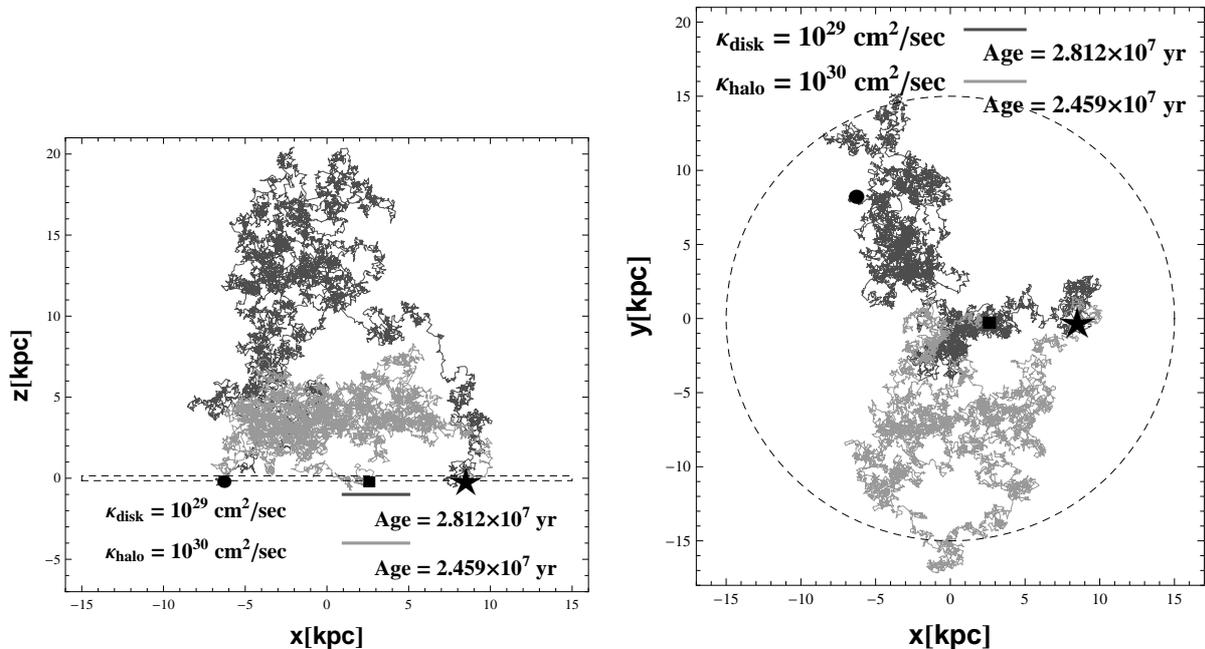


Fig. 5. Sample path for particles with $\kappa_{disk} = 10^{29} \text{ cm}^2 \text{ sec}^{-1}$ seen from the side (right) and top (left) of the Galaxy.

simulated particle. By this numerical experiment we can obtain the age and path length distributions for various population of particles simultaneously. In this study, we investigated for the three cases of $\kappa_{disk} = 10^{28} \text{ cm}^2 \text{ sec}^{-1}$, $10^{29} \text{ cm}^2 \text{ sec}^{-1}$ and $10^{30} \text{ cm}^2 \text{ sec}^{-1}$.

III. RESULTS AND DISCUSSIONS

Fig. 2 shows the CR age distributions calculated by our stochastic simulation method. The solid, dashed and dotted lines indicate the cases for particles with $\kappa_{disk} = 10^{28} \text{ cm}^2 \text{ sec}^{-1}$, $10^{29} \text{ cm}^2 \text{ sec}^{-1}$ and $10^{30} \text{ cm}^2 \text{ sec}^{-1}$, respectively. Here the CR age corresponded to the arrival time when the particles were traced backward in time from the Solar system to the SNR sources. We

find that the CR age is distributed over a wide range and the distribution and the position of the peak shift depending on the value of κ_{disk} ; the larger the value of κ_{disk} is, the distribution shifts to the smaller values of ages as expected for diffusion processes in general. This tendency suggests the energy dependence of the average age of GCRs which is implied by a positive correlation between the energy and the value of the diffusion coefficient of GCRs. We also found that the resultant distributions are represented well by lognormal distributions, indicated by the solid, dashed and dotted curves for each cases in fig. 2. Here the average values of the age distributions were 35.0 Myr, 6.72 Myr and

2.07 Myr for the cases of $\kappa_{disk} = 10^{28} \text{ cm}^2 \text{ sec}^{-1}$, $10^{29} \text{ cm}^2 \text{ sec}^{-1}$ and $10^{30} \text{ cm}^2 \text{ sec}^{-1}$, respectively.

The total path length was also calculated for the particles. The results are illustrated in fig. 3, where the solid, dashed and dotted lines indicate the cases for particles with $\kappa_{disk} = 10^{28} \text{ cm}^2 \text{ sec}^{-1}$, $10^{29} \text{ cm}^2 \text{ sec}^{-1}$ and $10^{30} \text{ cm}^2 \text{ sec}^{-1}$, respectively. It is seen that, as in the case of the CR age distribution shown in fig. 2, the total path length is also distributed over a wide range, and the distributions are energy dependent. The resultant distributions are also represented well by lognormal distributions shown by the curves in fig. 3. Here the averages of the total path lengths were 15.9 g cm^{-2} , 2.62 g cm^{-2} and 0.540 g cm^{-2} for the cases of $\kappa_{disk} = 10^{28} \text{ cm}^2 \text{ sec}^{-1}$, $10^{29} \text{ cm}^2 \text{ sec}^{-1}$ and $10^{30} \text{ cm}^2 \text{ sec}^{-1}$, respectively.

Distribution of CR sources in the Galaxy was also investigated by our stochastic simulation method. Fig. 4 show the distribution of positions of the sources of GCRs projected on $x - y$ plane, where left and right figures represent for particles with $\kappa_{disk} = 10^{28} \text{ cm}^2 \text{ sec}^{-1}$ and $10^{30} \text{ cm}^2 \text{ sec}^{-1}$, respectively. A black circle indicates the position of the Solar system and gray circles indicate the CR source. A large dashed circle indicates the boundary of the galactic disk. We found that the CR sources were distributed primarily in the local region around the Solar system and the distribution tends to extend over a larger region with increasing values of κ_{disk} , i.e., the CR energy.

Fig. 5 displays two sample paths of the particles from the SNR source to the Solar system with $\kappa_{disk} = 10^{29} \text{ cm}^2 \text{ sec}^{-1}$ projected on $x - z$ plane (left) and the same sample paths projected on $x - y$ plane (right), where the galactic disk is represented by a dashed line (left) and a large dashed circle (right). Black and gray curves indicate the calculated CR sample paths, which are connected from the solar system (star) to each SNR sources (circle and square). The particles propagated back and forth between the galactic disk and the outside region (halo). In particular, we found that the CRs spend long time outside of the galactic disk and that the trajectory is up to 20kpc away from the galactic disk in this example. These features revealed by sample paths are invaluable information about the propagation processes of GCRs not attainable easily by other numerical and analytical methods.

IV. SUMMARY

We have proposed a new stochastic method for the investigation of the propagation of GCRs which enables us to directly obtain the CR age, path length and source distributions as well as the sample path of CRs propagating in the Galaxy. The CR age and path length distribution were represented well by lognormal distributions and the average values of age and path length were consistent with that reported by other authors [4]. This method should be extended and investigated further for

more realistic model including the galactic wind, galactic magnetic field and energy losses.

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