Calculations of differential temperature coefficients for muons at different zenith angles (some practical aspects)

A.N. Dmitrieva, R.P. Kokoulin, A.A. Petrukhin and D.A. Timashkov

Moscow Engineering Physics Institute (State University), Moscow 115409, Russia

***Abstract*. Influence of atmospheric temperature on muon flux at sea level is considered. Results of calculations of the differential temperature coefficients (DTC) for muons at different zenith angles and threshold energies are presented. These calculations are based on formulas describing muon production and propagation in atmosphere. In calculations, a six-layer stationary spherical model of atmosphere is used, contributions of both pions and kaons are taken into account. Also for muons, relation between energy loss and muon energy is taken into account. Some practical questions of the use of DTC for muon hodoscope data processing are discussed.**

***Keywords*: cosmic rays, muon flux, temperature effect**

I. Definition and calculation of differential temperature coefficients

In investigations of muon flux variations at the Earth surface, barometric and temperature effects have to be taken into account. The creation of new type of ground-based detectors–muon hodoscopes–to measure spatial-angular variations of muon intensity requires development of methods of taking into account atmospheric corrections simultaneously and independently for all directions in the upper hemisphere. Besides, earlier calculations of this effects [1]-[2] were performed many years ago and demand critical re-consideration taking into account present knowledge of parameters of interactions of primary particles and secondary mesons and muons with air, and also modern models of the atmosphere. Present work and our previous paper [3] are devoted namely to these problems.

For calculation of temperature effect correction, it is necessary to know differential temperature coefficients, which make it possible to correct counting rate taking into account changes of the temperature at all altitudes of the atmosphere. Let us denote $N_0(E_{\min}, X, \theta)$ as integral muon intensity at observation level X (in atm) for zenith angle θ and threshold energy $E_{\min}$ at "standard" atmosphere. If atmospheric temperature is changed by $\Delta T(h)$ (h is the atmospheric depth in atm) muon flux will be changed by $\Delta N(E_{\min}, X, \theta)$ and one can write the relative change of the muon flux in a following way [1]:

$$\Delta N(E_{\min}, X, \theta)/N_0(E_{\min}, X, \theta)\cdot 100\% =$$
$$= \int_0^X W_{\mathrm{T}}(E_{\min}, X, h, \theta)\Delta T(h)dh \approx$$
$$\approx \sum_i W_{\mathrm{T}}(E_{\min}, X, h_i, \theta)\Delta T(h_i)\Delta h_i, \qquad (1)$$

here the function $W_{\mathrm{T}}(E_{\min}, X, h, \theta)$ is DTC.

If the altitude dependence of atmospheric temperature $T(h)$ and "standard" value of muon integral intensity are known, with the help of DTC it is possible to calculate the intensity corrected for the temperature effect N^{corr}:

$$N^{\mathrm{corr}}(E_{\min}, X, \theta) =$$
$$= N^{\mathrm{abs}}(E_{\min}, X, \theta) - \Delta N(E_{\min}, X, \theta), \qquad (2)$$

where N^{abs} is the measured muon integral intensity.

DTC can be found on the basis of the following formulas describing muon production and propagation in atmosphere [4]:

$$N(E_{\min}, X, \theta) = \int_{E_{\min}}^{\infty} dE \int_0^X dz \int_0^z dz'$$
$$\int_{\varepsilon}^{(\eta c^2/\mu c^2)^2 \varepsilon} dE_\eta F(E, X, \theta, z, z', E_\eta); \qquad (3)$$

$$F(E, X, \theta, z, z', E_\eta) = A_\eta \cdot \exp\left(-z'/L_p\right) \cdot E_\eta^{-\gamma} \cdot$$
$$\cdot \frac{B_\eta}{(1-(\mu c^2/\eta c^2)^2)E_\eta} \cdot \frac{d\varepsilon(E, X-z)}{dE} \cdot$$
$$\cdot \exp\left(-\frac{z-z'}{\lambda_\eta} - \frac{l(z)-l(z')}{c\tau_\eta}\cdot\frac{\eta c^2}{E_\eta}\right)\cdot$$
$$\cdot \frac{\eta c^2}{c\tau_\eta \rho(z) E_\eta} \cdot \exp\left(-\int_z^X \frac{\mu c^2 dt}{c\tau_\mu \varepsilon(E, X-t)\rho(t)}\right). \qquad (4)$$

Here E is muon energy at observation level X, E_η is the energy of produced meson (π or K), z and z' are depths of muon and meson generation along the track (in g/cm^2), ε is muon energy at depth z, η and μ are masses of meson and muon correspondingly, c is the velocity of light, τ_η and τ_μ are life-times of meson and muon correspondingly, A_η is the normalization constant ($A_K/A_\pi = 0.15$), L_p is the absorption length of primary nucleons in air, γ is the index of generation function of mesons (index of differential primary spectrum), λ_η is interaction mean free path of meson, ρ is air density, B_η is the probability of corresponding mode of two-body ($\mu\nu$) decay ($B_\pi = 1.0$, $B_K = 0.64$), $l(z) - l(z') = \int_{z'}^{z} dt/\rho(t)$.

In order to obtain the expression for $\Delta N(E_{\min}, X, \theta)$ it is necessary to vary the function $N(E_{\min}, X, \theta)$ with respect to temperature at a constant pressure P:

$$\Delta N(E_{\min},X,\theta)=\int_{E_{\min}}^{\infty}dE\int_0^X dz\int_0^z dz'$$
$$\int_{\varepsilon}^{(\eta c^2/\mu c^2)^2\varepsilon}dE_\eta F(E,X,\theta,z,z',E_\eta)\cdot$$
$$\cdot\left(\frac{\delta T(z)}{T(z)}-\frac{\eta c^2R}{c\tau_\eta E_\eta M}\int_{z'}^{z}\frac{\delta T(t)dt}{P(t)}-\right.$$
$$\left.-\frac{\mu c^2R}{c\tau_\mu M}\int_z^X\frac{\delta T(t)dt}{\varepsilon(E,X-t)P(t)}\right), \quad (5)$$

here M is molecular mass of air, R is universal gas constant. The variation factor in this formula is in agreement (up to dimensions of quantities) with formula presented by L.I.Dorman and V.G.Yanke [1]. First two terms in parentheses in the integral reflect the change of muon flux because of the change of pion and kaon decay probability; the third term reflects the variation of muon flux because of changes of muon decay probability. Thus, the temperature effect can be divided in two components (so-called meson effect and muon effect):

$$W_{\rm T}(E_{\min},X,h,\theta)=$$
$$=W_{\rm T}^{\eta}(E_{\min},X,h,\theta)+W_{\rm T}^{\mu}(E_{\min},X,h,\theta). \quad (6)$$

To obtain $W_{\rm T}(E_{\min},X,h,\theta)$ it is necessary to combine equations (1) and (5), then to take $\Delta T(h)$ equals to Dirac function $\delta(h)$. After transformation we obtain formulas for DTC calculation:

$$W_{\rm T}^{\eta}(E_{\min},X,h,\theta)=\frac{100\%}{N_0(E_{\min},X,\theta)}\cdot$$
$$\cdot\left[\frac{1}{T(t_0)}\int_{E_{\min}}^{\infty}dE\int_0^{t_0}dz'\int_{\varepsilon}^{(\eta^2/\mu^2)\varepsilon}FdE_\eta-\right.$$
$$-\int_{E_{\min}}^{\infty}dE\int_{t_0}^{X}dz\int_0^{t_0}dz'$$
$$\left.\int_{\varepsilon}^{(\eta^2/\mu^2)\varepsilon}dE_\eta F\cdot\left(\frac{\eta c^2R}{c\tau_\eta E_\eta MP(t_0)}\right)\right]; \quad (7)$$

$$W_{\rm T}^{\mu}(E_{\min},X,h,\theta)=\frac{100\%}{N_0(E_{\min},X,\theta)}\cdot$$
$$\int_{E_{\min}}^{\infty}dE\int_0^{t_0}dz\int_0^{z}dz'\int_{\varepsilon}^{(\eta^2/\mu^2)\varepsilon}dE_\eta F\cdot$$
$$\cdot\left(\frac{-\mu c^2R}{c\tau_\mu M\varepsilon(E,X-t_0)P(t_0)}\right). \quad (8)$$

The path t_0 (in g/cm^2) along the trajectory of the particle corresponds to the depth h.
The sign of the meson effect $W_{\rm T}^{\eta}$ is positive, since if the temperature of atmosphere increases, the atmosphere expands, density of air decreases and the probability of the interaction of mesons (kaons and pions) at unit of geometric path becomes smaller, hence relative probability of decay into muons becomes higher. Sign of the muon effect $W_{\rm T}^{\mu}$ is negative, because if the temperature of atmosphere increases and atmosphere expands, the geometric path from generation level to registration one becomes longer, so higher number of muons will decay. The relation of absolute values of the effects depends on $E_{\min}$. In case of low threshold energies the absolute value of the muon effect $W_{\rm T}^{\mu}$ is greater than the value of meson effect $W_{\rm T}^{\eta}$, and the sign of the total effect $W_{\rm T}$ is negative. In case of high threshold energies, the muon effect degrades (muons have no time to decay in the atmosphere) and the sign of the total effect becomes positive.

II. Models of atmosphere

For muon spectrum calculations, the dependence of air density on altitude above sea level has to be known. In our calculations, a six-layer stationary model of atmosphere [5] was used. In this model, the air is considered as an ideal gas. Ground surface and atmosphere are considered as spherical.
Dependence of air temperature on altitude above sea level for standard atmosphere is shown in Fig. 1 by solid line (AS designation). In the same figure, real measurements of temperature for Moscow region [6] (Russia) for February (the month of coldest temperatures) and June (the month of highest temperatures) of 2008 are shown by open and full symbols respectively. One can see from the figure that the difference between summer and winter may reach about 40 degrees. As examples of the use of calculated DTC (see calculations below) we decided to introduce additional models of atmosphere which cover the whole range of temperatures. These models are shown by additional lines in Fig. 1 and are designated as A1, A2, A3 and A4.

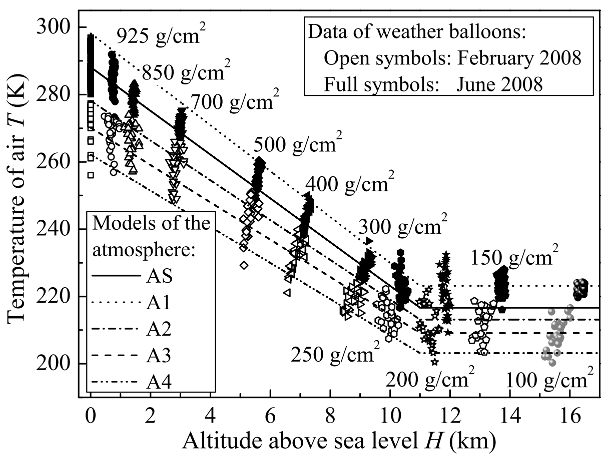

Fig. 1: Dependence of air temperature on altitude above sea level. Measurements of temperature (Moscow, Russia) for 2008 are shown by symbols. Calculations with models are shown by lines. AS is the standard model of atmosphere; A1, A2, A3 and A4 are additional models of atmosphere.

III. Results of DTC calculations

In our calculations the following values of parameters were used: $\gamma=2.7$, $L_p=110$ g/cm^2, $\lambda_\pi=120$ g/cm^2

and $\lambda_K = 150$ g/cm^2. DTCs were calculated for six values of zenith angle (0°, 15°, 30°, 45°, 60° and 75°) and for two types of detectors: muon telescopes (threshold energy does not depend on zenith angle $E_{\min} = \text{const}$) and muon hodoscopes (threshold energy usually depends on zenith angle as $E_{\min} = \text{const}/\cos\theta$). The value of "const" 0.4 GeV is close to the threshold energy of existing muon hodoscopes and we will consider correction for temperature effect for this energy. Total effect W_{T} for $E_{\min} = 0.4$ GeV is presented in Fig. 2 and Table I. For comparison, total effect W_{T} for $E_{\min} = 0.4$ GeV$/\cos\theta$ is presented in Fig. 3 and Table II. Examples of dependence of DTC on atmospheric depth h for $\theta = 0°$ and six values of threshold energy are shown in Fig. 4. From this figure one can see that for threshold energies above 10 GeV DTC becomes positive in significant part of atmosphere depth.
Comparison of results of our DTC calculations with results of earlier works was disscussed in paper [3]. It exhibited only qualitative agreement with the preceding results, whereas quantitative differences amounted to tens percent. These differences can be caused by approximations and insufficient precision of numerical calculations in earlier works.

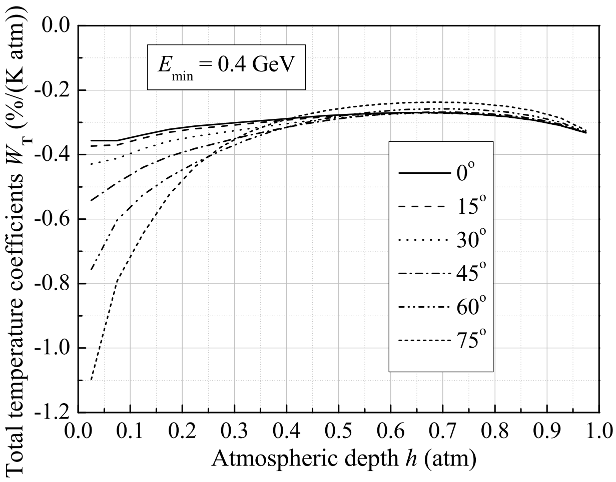

Fig. 2: Total differential temperature coefficients W_{T} calculated for $E_{\min} = 0.4$ GeV.

As an example of the use of our DTC, integral muon intensity was calculated directly by formulae (3) for five types of atmospheric temperature profile (AS, A1, A2, A3 and A4, see the description of atmospheric models above). Results of calculations for hodoscopes with $E_{\min} = 0.4$ GeV$/\cos\theta$ are presented in Fig. 5. One can see that for higher temperature the intensity is lower, and vice versa. Difference between calculated integral muon intensity $N(E_{\min}, X, \theta)$ and value $N_0(E_{\min}, X, \theta)$ for standard atmosphere (AS) is shown in Fig. 6 by full symbols. One can see that it reaches about 8 % and slightly depends on zenith angle. After correction of muon counting rate for temperature effect using formulae (2) (open symbols in Fig. 6) the difference of results for various models of atmosphere does not exceed 0.3 %.

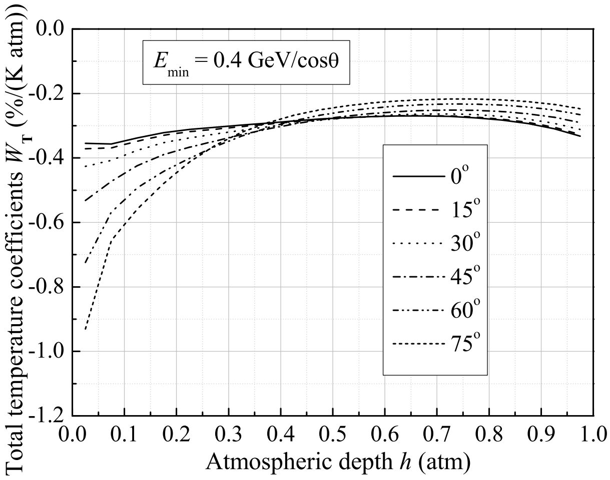

Fig. 3: Total differential temperature coefficients W_{T} calculated for $E_{\min} = 0.4$ GeV$/\cos\theta$.

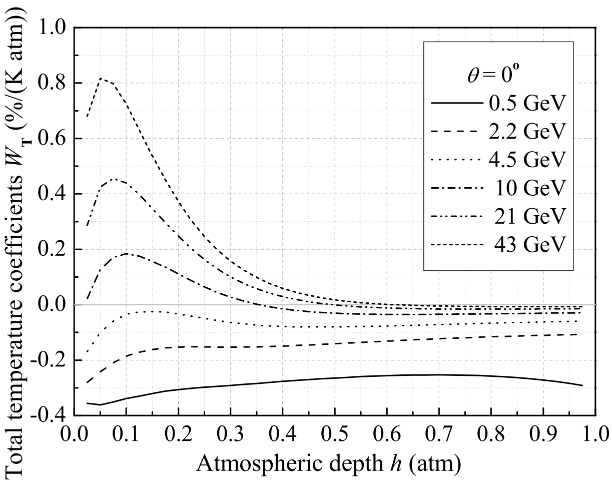

Fig. 4: Total differential temperature coefficients W_{T} calculated for $\theta = 0°$ and several values of threshold energy.

The non-zero value of this difference is explained by uncertainties which are caused by the use of summation instead of exact integral (see formulae (1)). Calculations show that for telescopes with $E_{\min} = 0.4$ GeV the precision of corrections is similar.

IV. Conclusion

Differential temperature coefficients were calculated for updated parameters of interactions of primary and secondary particles with air, for six-layer spherical model of atmosphere, taking into account contributions of both pions and kaons to muon flux. Results of calculations for several values of zenith angle and threshold energy are presented. It is shown that the use of calculated DTC for $E_{\min} = 0.4$ GeV$/\cos\theta$ for temperature effect decreases the difference between the corrected and standard value of integral muon intensity to less than 0.3 %.

TABLE I: Total differential temperature coefficients for $E_{\min} = 0.4$ GeV.

h, atm	W_T, % / (K atm) 0°	15°	30°	45°	60°	75°
0.025	-0.356	-0.373	-0.429	-0.542	-0.756	-1.140
0.075	-0.357	-0.370	-0.412	-0.487	-0.604	-0.780
0.125	-0.338	-0.349	-0.382	-0.439	-0.524	-0.639
0.175	-0.322	-0.331	-0.358	-0.405	-0.469	-0.522
0.225	-0.312	-0.320	-0.343	-0.381	-0.426	-0.435
0.275	-0.305	-0.312	-0.332	-0.361	-0.387	-0.374
0.325	-0.298	-0.304	-0.320	-0.341	-0.354	-0.332
0.375	-0.292	-0.297	-0.309	-0.323	-0.326	-0.301
0.425	-0.285	-0.289	-0.299	-0.308	-0.305	-0.279
0.475	-0.279	-0.282	-0.289	-0.294	-0.288	-0.263
0.525	-0.275	-0.277	-0.282	-0.284	-0.275	-0.252
0.575	-0.271	-0.273	-0.276	-0.276	-0.266	-0.244
0.625	-0.270	-0.271	-0.273	-0.271	-0.260	-0.239
0.675	-0.269	-0.270	-0.271	-0.268	-0.258	-0.237
0.725	-0.271	-0.272	-0.272	-0.269	-0.258	-0.238
0.775	-0.275	-0.276	-0.275	-0.272	-0.261	-0.242
0.825	-0.283	-0.283	-0.282	-0.278	-0.268	-0.250
0.875	-0.293	-0.293	-0.293	-0.289	-0.280	-0.264
0.925	-0.309	-0.309	-0.308	-0.305	-0.299	-0.286
0.975	-0.332	-0.332	-0.332	-0.332	-0.330	-0.325

TABLE II: Total differential temperature coefficients for $E_{\min} = 0.4\ \mathrm{GeV}/\cos\theta$.

h, atm	W_T, % / (K atm) 0°	15°	30°	45°	60°	75°
0.025	-0.356	-0.372	-0.426	-0.532	-0.724	-1.060
0.075	-0.357	-0.369	-0.407	-0.473	-0.568	-0.717
0.125	-0.338	-0.348	-0.376	-0.424	-0.492	-0.595
0.175	-0.322	-0.329	-0.353	-0.390	-0.442	-0.487
0.225	-0.312	-0.318	-0.338	-0.367	-0.401	-0.405
0.275	-0.305	-0.311	-0.326	-0.347	-0.364	-0.348
0.325	-0.298	-0.303	-0.314	-0.328	-0.332	-0.307
0.375	-0.292	-0.295	-0.303	-0.310	-0.306	-0.278
0.425	-0.285	-0.288	-0.293	-0.294	-0.285	-0.256
0.475	-0.279	-0.281	-0.283	-0.281	-0.268	-0.240
0.525	-0.275	-0.276	-0.276	-0.270	-0.255	-0.229
0.575	-0.271	-0.271	-0.270	-0.262	-0.245	-0.220
0.625	-0.270	-0.269	-0.266	-0.256	-0.239	-0.214
0.675	-0.269	-0.268	-0.264	-0.253	-0.234	-0.210
0.725	-0.271	-0.270	-0.264	-0.251	-0.233	-0.208
0.775	-0.275	-0.273	-0.266	-0.252	-0.233	-0.208
0.825	-0.283	-0.280	-0.271	-0.256	-0.236	-0.211
0.875	-0.293	-0.290	-0.280	-0.263	-0.242	-0.216
0.925	-0.309	-0.305	-0.293	-0.274	-0.251	-0.225
0.975	-0.332	-0.327	-0.312	-0.290	-0.265	-0.238

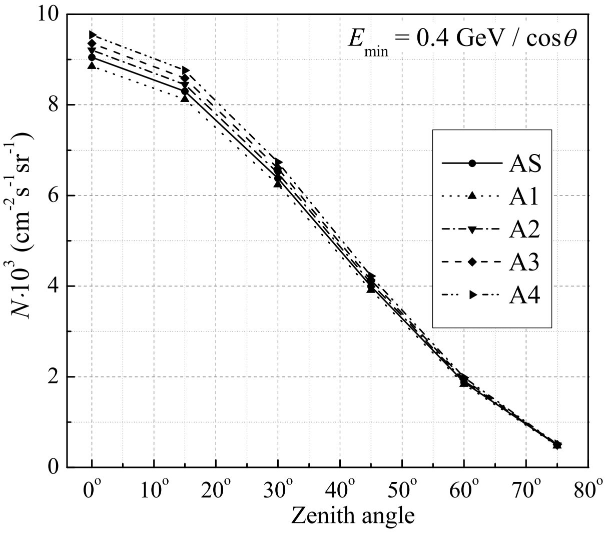

Fig. 5: Integral muon intensity calculated for five types of atmospheric temperature profile (AS, A1, A2, A3 and A4, see above).

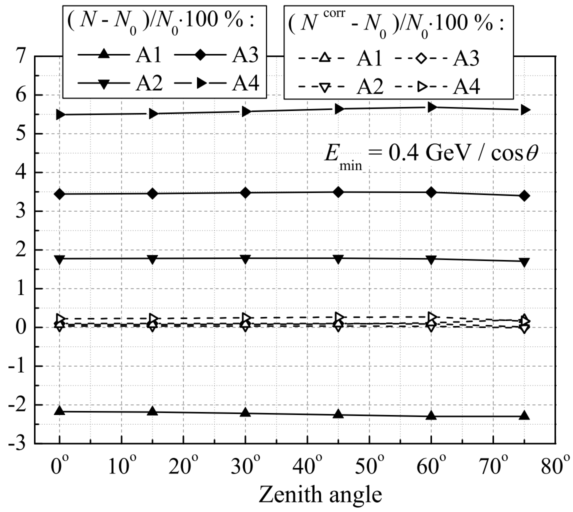

Fig. 6: Difference between calculated integral muon intensity and value for standard atmosphere (AS). Differences without correction for temperature effect are shown by full symbols, after correction by open symbols. A1, A2, A3 and A4 are atmospheric models.

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REFERENCES

[1] L. I. Dorman, Meteorological effects of cosmic rays. Moscow: Nauka, 1972 (in Russian).
[2] K. Maeda, J. Atm. Terr. Phys. 19, No 3-4 (1960) 184-245.
[3] A. N. Dmitrieva et al. "Calculation of differential temperature coefficients for muons at different zenith angles", http://ecrs2008.saske.sk/dvd/s1.09.pdf
[4] L. V. Volkova, Calculation of fluxes and angular distributions of high energy atmospheric muons at sea level. Preprint FIAN SSSR No 72, Moscow, 1969 (in Russian).
[5] Yu. A. Glagolev, Reference book on physical parameters of the atmosphere. Leningrad: Gidrometeoizdat, 1970 (in Russian).
[6] Central Aerological Observatory (Russia, Dolgoprudny), http://www.cao-rhms.ru/index.html