Implementation of a muon lifetime experiment in an undergraduate laboratory course at the Colorado School of Mines

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***Abstract*. In this well-known muon lifetime experiment, a fundamental property of a fundamental particle can be measured using a vessel of liquid scintillator viewed by a single photomultiplier tube. This poster describes the implementation of this experiment for an advanced undergraduate laboratory course in which student teams have two weeks to perform the experiment and write a report. The apparatus construction and drafting of instructions for the course were accomplished by two undergraduates as part of their senior design project. The effectiveness of this experiment in a laboratory course will also be presented.**

***Keywords*: muon lifetime teaching**

I. Motivation

The muon is a fundamental particle in the Standard Model having the primary decay modes of

$$\mu^- \rightarrow e^- + \overline{\nu}_{e^-} + \nu_{\mu^-},$$
$$\mu^+ \rightarrow e^+ + \overline{\nu}_{e^+} + \nu_{\mu^+}.$$

The muon was discovered in the cosmic ray cloud chamber work of Anderson and Neddermeyer in 1937. Three years later its lifetime was measured by Rossi and D. Hall at Echo Lake and Denver, Colorado. Providing an early test of relavistic time-dilation, their result of 2.3 $\pm$ 0.2 μs [2] is consistent with the current world average of 2.197019(21) μs, determined by accelerator-based experiments[3]. The experiment described here follows the simplified design developed by R. Hall, Lind and Ristinen[4] in 1970. Their design requiring a single photomultiplier tube (PMT) has become a well-known and elegant classic[5]. It offers an outstanding hands-on opportunity for students to combine their laboratory skills with their theoretical understanding of modern physics to measure a fundamental property of a fundamental particle.

This experiment was constructed as a Senior Design (SD) project at the Colorado School of Mines (CSM). Each CSM physics major conducts a SD project in their senior year working with a faculty mentor. This particular project of authors Bowles and Prowers was to build the experiment, measure the muon lifetime, draft a set of instructions, and finally implement the experiment in an undergraduate laboratory course. In this course, PHGN 326, students perform five experiments in nuclear and cosmic-ray physics over the semester working in teams of two or three. 61 students, primarily 3rd year physics majors, were enrolled in PHGN 326 in the (northern) spring 2009 semester when this experiment was introduced. 9 teams (25 students) set up the apparatus and measured the muon lifetime.

II. The Apparatus

The schematic of the experiment accompanied by typical oscilloscope traces is shown in figure 1. When muons pass through a vessel of liquid scintillator, they deposit kinetic energy by ionization. Some of the light produced is detected by the PMT. Most muons have an energy near 1 GeV and travel completely through the detector. However, if a low-energy muon is *stopped* within the detector, it decays within a few lifetimes to an electron, neutrino, and antineutrino. The neutrino interaction cross sections are too small to interact with the scintillator, but the electron deposits enough energy to generate a second light pulse. Hence, the signature of muon decay is a PMT pulse pair separated on the time scale of a muon lifetime.

The detector consists of a 20 gallon aluminum pot of liquid scintillator and an attached 3” RCA PMT. An aluminum plate was machined to form a light-tight lid secured by two draw hasps. The PMT was mounted in a plastic fixture arranged so the PMT face extended below the surface of the scintillator. The EJ-321-P organic mineral-oil-based liquid scintillator from Eljen Technology has a peak output of 425 nm. An Ortec 454 timing amplifier is used to amplify the PMT output. A 100 MHz Ortec 436 leading edge discriminator identifies scintillation peaks, and the TAC is an Ortec 567. An Ortec PC multichannel analyzer (2048 bins) card and software histogram the data.

The lifetime measurement requires muons of sufficiently low energy to stop within the detector. Scintillators have an energy absorption rate proportional to the distance the particles travel. This rate is about 1.7-2 MeV/cm for plastic scintillator. Scaling by the relative density of plastic and liquid scintillator and multiplying by the 39 cm scintillator depth yields 55 MeV deposited per vertical muon. The expected PMT pulse rate, ie the rate of muons hitting the detector, was estimated to be 60 Hz, based on a muon flux of $2/(cm^2 \cdot min)$ [4] at the 1800 m altitude of Golden CO.

Many of the PMT pulses were as small as 5 mV, so an amplifier was needed to drive them above the minimum discriminator threshold of 50 mV. The amplifier requires time resolution on the order of 0.1 μs. An amplification of 10 prevented the average PMT line noise of $\pm$2.5

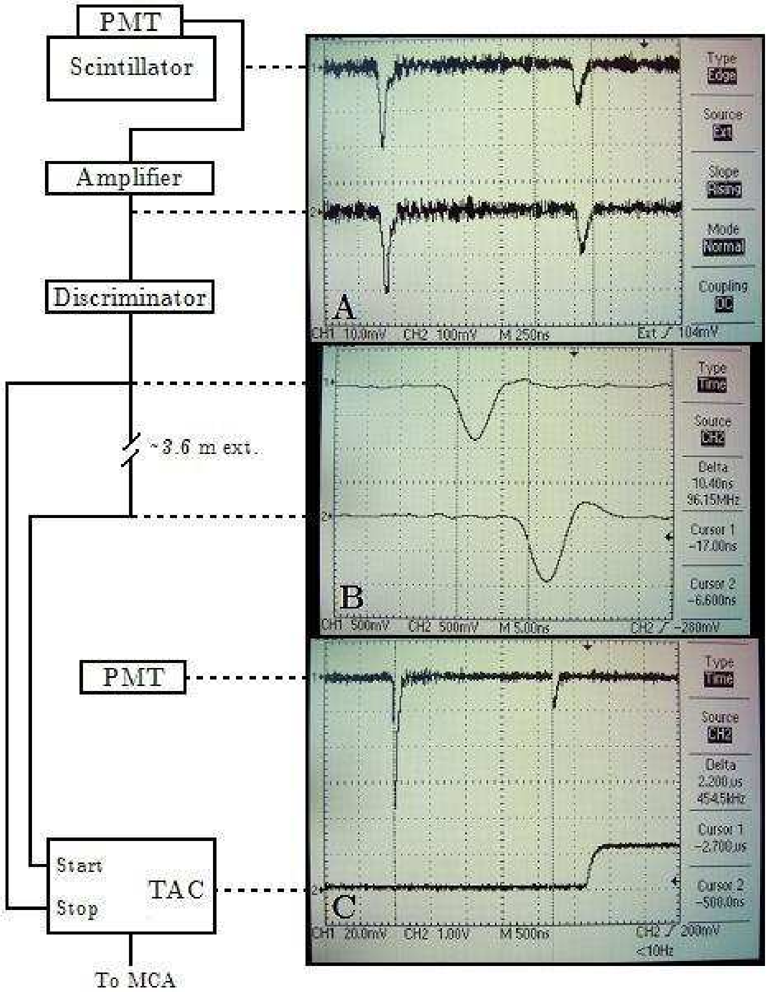

Fig. 1: Left: Schematic of selected experiment. Right: Actual measured voltage of **A:** PMT and amplified PMT pulse pair, **B:** TAC stop and start inputs, **C:** TAC output with PMT pulse pair. The oscilloscope readings are to illustrate PMT amplification, the delay in the start pulse sent to the TAC, and a typical calibration measurement.

mV from triggering the discriminator. By adjusting the threshold to 100 mV, we obtained a discriminator trigger rate of 50 Hz, in rough agreement with the expected rate R. The discriminator output has typical rise and fall times of 4.4 ns and 4.5 ns, respectively which are much shorter than the muon lifetime.

In its ready state, the Time to Amplitude Converter (TAC) will only accept a start pulse and ignores stray stop pulses. Once a valid start pulse is received, the TAC enters a timing state that is ended either by a stop pulse or the TAC resetting. The TAC resets and produces no output if a stop pulse is not received within the user-set 20 μs timing range.

The start pulse to the TAC is delayed approximately 10 ns seconds by a 3.7 m cable. When a scintillation pulse occurs and the discriminator is triggered, the initial stop pulse is ignored because it arrives slightly before the start pulse. Because the discriminator pulse is short, this stop pulse returns low before the start pulse arrives about 4 ns later. The TAC then begins timing upon the arrival of the start pulse. When a muon decays within the scintillator, a second pulse is generated that arrives immediately at the stop input to the TAC. The size of the TAC output pulse is proportional to the time between the start and stop pulses and represents the decay time of the muon minus the start cable delay. Because the TAC does not accept input while in its 2 μs output cycle, the arrival of the start pulse 4 ns after the valid stop pulse is ignored. The TAC output voltage pulse is then recorded by a Multi-Channel Analyzer (MCA).

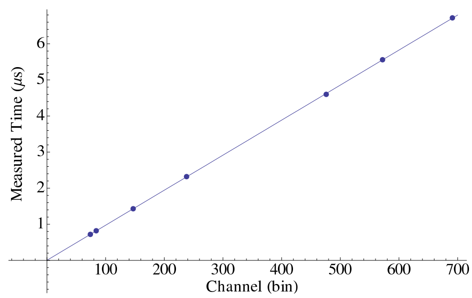

Fig. 2: Time to MCA channel calibration example. Each point represents a muon decay.

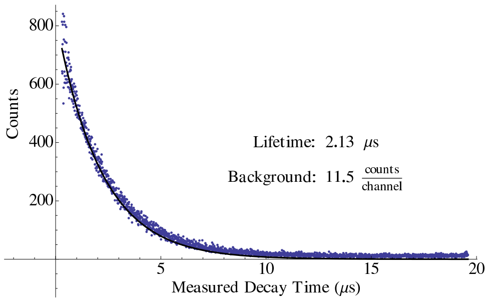

Fig. 3: Exponential fit.

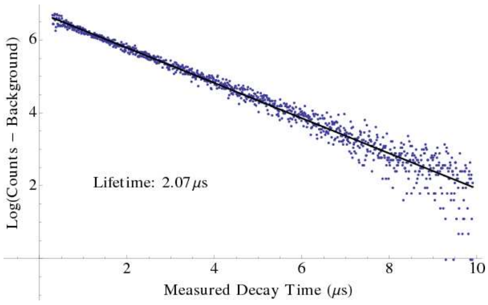

Fig. 4: Linear Fit to Logarithm

The calibration constant between MCA bins and time was determined by examining a small number of muon decay measurements. Triggering individual muon decays with the TAC, allowed by-hand measurement of PMT pulse pairs time separation on the oscilloscope and the recording of the corresponding MCA bin. (See Figure 2). Least squares linear regression was used to find the best fit line $t = \hat{m}b + \hat{t}_0$ where m is the conversion constant.

III. Summary of Analysis and Results

The form of the spectral data is thus a combination of exponential decay and constant background, and is well described by $N(t) = N_0 e^{-\lambda t} + C$ where $N(t)$ is the number of counts in the bin corresponding to time t, C is the background rate in counts per channel, and N_0 is a fit parameter.

The muon lifetime results were obtained from three analysis methods. Figure 3 is a fit of the exponential form. The fit line follows the spectral data well for shorter lifetime measurements, but loses accuracy in the background region. The lifetime obtained using this method is 2.13 μs. Figure 4 displays a line fit to the logarithm of the spectral data minus the background counts. Notice the growing variance in the data for longer lifetime measurements. The calculated muon lifetime for this method is 2.07 μs.

Finally, Figure 5 displays the results obtained using the Method of Moments as outlined below:

- Estimate τ_μ by solving $E[T] = \overline{T}$
  - $\overline{T}$ is the average measured time
- Expectation value $E[T] = \displaystyle\int_{-\infty}^{\infty} t\, p(t)\, dt$

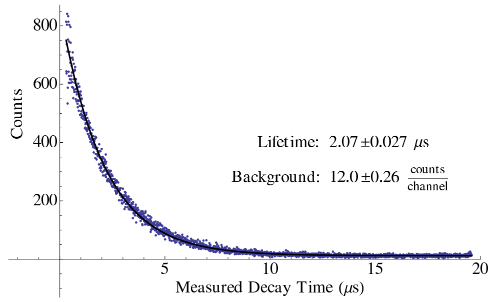

Fig. 5: Method of Moments fit

- $p(t)$ is measurement probability density function
  - Superposition of uniform and truncated exponential distributions:

    $$p(t) = \alpha \frac{1}{t_h - t_\ell} + (1-\alpha)\frac{\lambda e^{-\lambda t}}{\int_{t_\ell}^{t_h} \lambda e^{-\lambda t}}, t_\ell < t < t_h$$

    $p(t) = 0$ otherwise
  - t is time, with $t = 0$ when muon is incident
  - t_ℓ and t_h are lower and upper bounds on measurement range
  - α is the probability that a given measurement is background estimated as the ratio of total background counts to total counts
  - Measurements longer than 12.62 μs considered background
- Solve $E[T] = \overline{T}$ for τ_μ:

  $$\tau_\mu = \frac{1}{\lambda} = \frac{\overline{T} - \frac{1}{2}\alpha(t_h + t_\ell)}{1 - \alpha} - t_\ell$$
- Uncertainties in $\overline{T}$ and background Normally distributed

The muon lifetime obtained from the Method of Moments is 2.07 μs.

The muon lifetime was also determined at a 95% confidence interval for the 5 data sets using the Method of Moments. Table I summarizes the findings. The first 3 data sets were collected with a smaller prototype detector, and data sets 4 and 5 were collected using the new detector. All results are lower than the accepted muon lifetime. There is likely a systematic error. The leading candidate is radiative muon capture that affects negative muons, but not positive muons. Negative muons can be captured into atomic orbitals and may interact weakly with the nucleus *before* they decay which may drive down the overall measurement average.

TABLE I: A summary of the muon lifetimes obtained using the Method of Moments for 5 data sets.

Data Set	τ (μs)
1	2.00±0.052
2	2.02±0.037
3	2.04±0.043
4	2.13±0.044
5	2.07±0.027

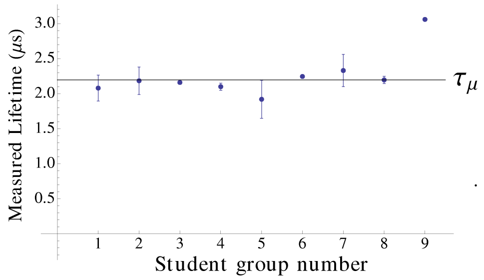

Fig. 6: Lifetime measurements of 9 PHGN326 student teams.

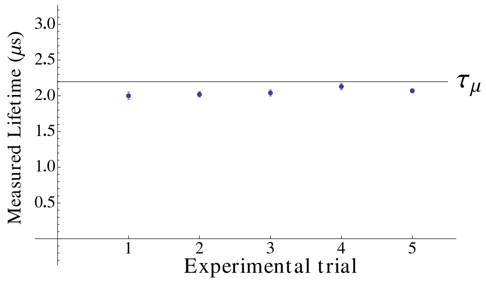

Fig. 7: Lifetime measurements by Senior Design team.

IV. Comments on Implementation in PHGN326

Each of the 9 teams were able to set up the experiment within their day-long lab session, confirm that it was accumulating muon decays, and collect an overnight data set of about 25,000 points, sufficient to measure the muon lifetime to about 10%. Each team then had two weeks to prepare a laboratory report detailing their findings.

Most students found the experiment interesting and their understanding of the measurement process was reasonable – they understood what a pulse pair represented. Also, each team performed the calibration step fairly easily, which reinforced their understanding of the experiment. There was however some confusion about what was occurring inside the scintillator and what the background represented. The biggest complications were found in the data and error analysis, most commonly in the estimation and subtraction of the background. The students' results (Figure 6) were, on average, closer to the established value than those obtained through more rigorous measurements (Figure 7) of the Senior Design team (Figure 8) . Possible reasons will not be discussed here, but this effect will be addressed when the course is offered in 2010. Resources have been identified for a second apparatus to give all future students enrolled in the course the opportunity to measure the muon lifetime.

V. Cost Estimate

A detailed parts list for the detector and electronics is provided in Table II. Many of the components used are obsolete, so equivalent components are quoted. The cables are all RG 58 A/U BNC. The necessary lengths are four 1 ft cables, one 2 ft cable, three 8 ft cables, and one 12 ft cable. A 50 Ω BNC terminator and 4 tees are also needed. Ortec includes the MCA software with the purchase of an MCA card. The price for the liquid scintillator is for 54 L at $14.50/L. The hardware for the pot and lid includes foam, putty, and two hasps. .

TABLE II: Parts list and costs (Jan. 2009) for reproducing the experiment from new components.

Part	Supplier	Price ($)
4001A NIM Bin	Ortec	1006
3002D HVPS	Canberra	2105
474 Timing Amplifier	Ortec	1521
584 Discriminator	Ortec	1530
566 TAC	Ortec	1722
TRUMP-PCI MCA Card	Ortec	3204
Maestro-32 MCA Software	Ortec	0
Cables and Fittings	Ortec	50
TDS2012B Oscilloscope	Ortec	1449
EJ-321P Liquid Scintillator	Eljen	780
PMT and Base	N/A	800
Computer (Optiplex 360)	Dell	333
20 Gal. Aluminum Pot	Kitchen Supply	70
2 x 2' x 3/8" 6061-T6 Aluminum Plate for Lid	Discountsteel.com	100
4" Dia. Acetal (hold PMT)	Onlinemetals.com	66
Hardware for Pot and Lid	Local	23
		$14759

Fig. 8: J. Prowers and M. Bowles with the detector

VI. Acknowledgments

We acknowledge the work of Prof. Uwe Griefe who revamped the advanced lab course and initiated this experiment. The assistance of the course's head TA, David Schuster, and that of department staff members Orlen Wolf and Larry Peterson is greatly appreciated.

References

[1] S. Neddermeyer and C. Anderson, "Note on the Nature of Cosmic-Ray Particles," Physical Review, **51**, 884-886, 1937.

[2] B. Rossi and D. B. Hall, "Variation of the Rate of Decay of Mesotrons with Momentum," Physical Review **59** (3), 223-228 (1941).

[3] D. B. Chitwood et. al., "Improved Measurement of the Positive-Muon Lifetime and Determination of the Fermi Constant," Physical Review Letters **99** (032001), 1-5, (2007).

[4] R. E. Hall, D. A. Ristinen, "A Simplified Muon Lifetime Experiment for the Instructional Laboratory," American Journal of Physics **38** (10), 1196-1200 (1970).

[5] A. C. Melissinos, J Napolitano, "Experiments in Modern Physics," 2nd Ed., © 2003 by Academic Press.