Modeling coherent geomagnetic radiation from cosmic ray induced air showers

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***Abstract*.** **An Ultra High Enegy Cosmic Ray (UHECR) entering the Earth's atmosphere will induce an Extensive Air Shower. This can be visualized by a 'pancake' of particles flying toward the Earth's surface with approximately the speed of light. The shower front will contain many charged particles. The deflection of these charged particles in combination with retardation effects will cause an electromagnetic wave to be emitted within the radio frequency range. For observer distances close to the impact point of the shower, the typical shape of this pulse will be determined mainly by the longitudinal distribution of the charged particles within the shower front, defined by our 'pancake' function. For observers placed further away from the impact point the longitudinal distribution for the total number of charged particles over time, the shower profile, becomes the main parameter. This model has been extended for realistic shower simulations, also including several sub-leading contributions. Inclusion of sub-leading contributions shows that the polarization of the radio signal becomes observer position dependent.**

***Keywords*: Air Shower simulations, Coherent radio emission, Geo-magnetic**

I. Introduction

In recent years interest in detection of cosmic ray air showers with the use of radio antennas has increased significantly. This is due to the promising results obtained by the LOPES [1], [2], and CODALEMA [3] experiments. These results triggered plans for an extensive array of radio detectors at the Pierre Auger observatory [4].

Several different models describing the radio signal coming from an air shower have been developed during the last few years. A microscopic description in terms of synchrotron emission of electrons and positrons gyrating in the Earth magnetic field is given in Ref. [5], [6], [7], [8]. In the earliest work on radio emission from air showers [9], [10], [11], [12], the importance of coherent emission has already been pointed out. Recently this approach has been extended for realistic simulations [13], [14]. The importance of different length scales within the air shower for this model is stressed out in [15]. Furthermore the influence of a realistic index of refraction has been tested [14]. The main subject of this article will be the dependence of the pulse shape on the observer position. We will also look into polarization effects for the largest subleading contribution which is due to charge excess. For simplicity, we will consider a perpendicular incoming air shower with velocity $\vec{\beta_s} = -\beta\hat{z}$, the magnetic field is considered to be perpendicular to the incoming air shower $\vec{H} = H\hat{y}$.

II. The Model

The collision of an UHECR with the Earth's atmosphere will cause an air shower. This can be visualized by a 'pancake' of particles flying toward the Earth's surface. The radio pulse coming from such an air shower can be described in a macroscopic picture by the Macroscopic Geomagnetic Radiation Model, as done in Refs. [13], [14]. As stressed in Ref. [15], there are several important length scales to be considered. One of these length scales is the typical longitudinal length of the pancake, given by L. The longitudinal position of electrons and positron at a fixed shower time t' within the pancake is given by h. The other important length scale is the distance d defined to be the shortest distance from the impact point of the air shower to the observer position, also called the impact parameter.

A. Geo-magnetic radiation

The deflection of electrons and positrons due to a lorentz force acting on them in the Earth magnetic field will cause a net current in the air. The total number of electrons and positrons in the shower at a fixed height defined by $z = -\beta_s t' + h$, where t' is the shower time defined such that at $t' = 0$ the showerfront hits the Earth, is now given by,

$$N_e(z, t') = N_e f_t(t') f(h) \ , \tag{1}$$

where N_e is total number of particles at the shower maximum, $f_t(t')$ is the normalized shower profile, and $f(h)$ gives the 'pancake' function describing the longitudinal spread of the charged particles from the shower front.

Macroscopically, for the given geometry a net current in the $\hat{x}$ direction can now be defined,

$$j(\vec{x}, z, t') = \langle v_d q \rangle e N_e(z, t') \delta(x) \delta(y) \ , \tag{2}$$

where q is the electric charge and where the lateral spread of the charged particles in the shower is neglected by concentrating them onto the shower axis using delta functions $\delta(x)$, and $\delta(y)$.

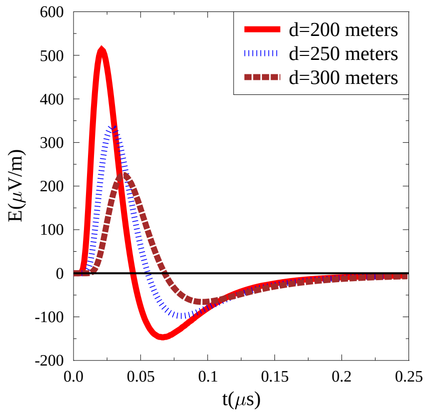


Fig. 1: The electric field for different observer distances from the impact point.

The electric field can now be obtained using the Liénard-Wiechert potentials from classical electrodynamics [16],

$$A^x(t,\vec{d}) = J\int \left.\frac{f_t(t_r)f(h)}{R(1-\vec{\beta}\cdot\hat{n})}\right|_{t'=t_r} dh\ , \quad (3)$$

where $J = \langle v_d q\rangle eN_e/4\pi\epsilon_0$. The electric field is evaluated at the retarded time $t' = t_r$ defined by $t - t_r \equiv R$. We can also define a retarded distance by $\mathcal{D} \equiv R(1 - \vec{\beta}\cdot\hat{n})$. Now writing $R = \sqrt{z^2+d^2}$, we can link the retarded time t_r to the observer time t, and the position from the shower front from where the signal is emitted, h,

$$\begin{aligned} t_r &= \frac{1}{1-n^2\beta^2}\Big[t - n^2\beta h \\ &- n\sqrt{(-\beta t + h)^2 + (1-n^2\beta^2)d^2}\Big]\ . \end{aligned} \quad (4)$$

For large impact parameters, we can now make a good approximation for the vector potential. By leaving out the integration over the pancake and defining $f(h) = \delta(h)$ putting $h = 0$, taking the limits $\beta_s = 1$ and for the index of refraction $n = 1$ now give,

$$\begin{aligned} \mathcal{D} &= R(1-\vec{\beta_s}\cdot\hat{n})|_{t'=t_r} \\ &= c\beta_s t + \mathcal{O}(1-\beta_s^2) \approx ct \end{aligned} \quad (5)$$

and

$$ct_r = \frac{ct}{1+\beta_s} - \frac{d^2}{2c\beta_s t} + \mathcal{O}(1-\beta_s^2) \approx -\frac{d^2}{2ct}\ . \quad (6)$$

The vector potential for large impact parameters now becomes,

$$A^x(t,\vec{d}) = J\left.\frac{f_t(t')}{\mathcal{D}}\right|_{t'=t_r}\ . \quad (7)$$

In Eq. 3, the integration is performed over the longitudinal position in the pancake, h. So the signal observed at an observer time t, from an position h within the pancake is emitted at the retarded shower time determined by Eq. 4. However for an observer placed close to the shower core, it is more transparent to change the integration variable and integrate over the retarded shower time at which the signal is emitted since the signal travels with the same velocity as the shower. The vector potential for small impact parameters can thus be written as,

$$\begin{aligned} A^x(t,\vec{d}) &= J\int dt_r \frac{f_t(t_r)}{\mathcal{D}}\frac{\partial h}{\partial t_r}f(h) \\ &= J\int dt_r \frac{f_t(t_r)}{\mathcal{D}}\frac{-\mathcal{D}}{z}f(h) \\ &= -J\int dt_r \frac{f_t(t_r)}{z}f(h)\ . \end{aligned} \quad (8)$$

The electric field can now be obtained by taking the derivative of the vector potential. For large impact parameters a good analytic approximation for the electric field is now obtained,

$$E_x(t,\vec{d}) \approx J\frac{c^2t_r^2 4}{d^4}\frac{d}{dt_r}[t_r f_t(t_r)]\ . \quad (9)$$

The electric field for small impact parameters can now be written as,

$$E^x(t,\vec{d}) = -J\int_{t_r^-(t)}^{t_r^+(t)} dt_r \frac{f_t(t_r)}{z}c\beta_s\frac{df(h)}{dh}, \quad (10)$$

where the limits are $t_r^- = -\infty$, and $t_r^+ = t_r(t, h=0)$. Note t, t_r, and h are linked to each other by Eq. 4.

It follows that the pulse shape for an observer placed far away from the impact point is determined mainly by the shower profile function $f_t(t_r)$, whereas the pulse shape for an observer placed close the the impact point scales approximately with $\frac{df(h)}{dh}$, the derivative of the longitudinal distribution of the particles inside the pancake. In Fig. 1, the electric field is plotted as function of the observer time for different observer distances. As expected the pulse height falls off, and the pulse itself becomes wider for observers placed further away from the impact point.

The integral in Eq. 8 has a divergency at $z = 0$ due to the fact that the total number of particles is still finite when the shower reaches the Earth. We can regularize this divergency by introducing an exponential cut-off for the total number of particles reaching the Earth such that the normalized shower profile goes to zero linearly for $z = 0$. The total normalized shower profile now becomes,

$$F_t(t_r) = f_t(t_r) - f_t(0)e^{-z/a}\ , \quad (11)$$

where a is a typical cut-off height. In Fig. 2 the electric pulse is plotted for different values of the cut-off parameter a. We can conclude that for a distance of $d = 50$ m from the impact point the pulse height becomes independent of the cut-off parameter a. This can also be seen in Fig. 3 showing the lateral distribution

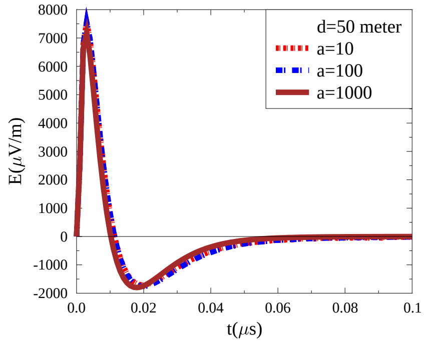


Fig. 2: The electric field seen by an observer placed at $d = 50$ meters from the impact point for different cut-off parameters a.

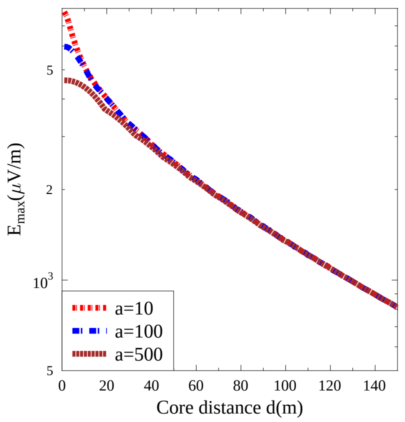


Fig. 3: The lateral distribution function for different values of the impact parameter a. The maximum pulse height of the electric field is plotted as function of the observer distance d.

function for different values of the cut-off parameter a. It follows that for impact parameters larger than $d \approx 30$ m we can safely introduce a cut-off term in the shower profile.

B. Charge Excess

Due to knock out effects from air molecules there will be a net charge excess of about 20% of the total number of electrons and positrons in the shower [17], [18]. Therefore a net negative charge will move toward the Earth with the shower. At the same time electrons will loose velocity and drop out of the pancake staying behind and contribute to the zero'th component of the vector potential. The total vector potential for the charge excess is thus given by,

$$A^0_{Cx} = \frac{eN_e}{4\pi\epsilon_0} C_x \frac{f_t}{D} \tag{12}$$

$$A^z_{Cx} = -\beta A^0_{Cx} = \frac{eN_e}{4\pi\epsilon_0} C_x \frac{-\beta f_t}{D}. \tag{13}$$

Note that $A^z_{Cx} = -\beta A^0_{Cx}$ due to the fact that the electrons in the shower front are moving with a velocity $\vec{\beta} = -\beta\hat{z}$. The electric field is now obtained by,

$$E^{Cx}_x(t) = \frac{-\partial A^0_{Cx}}{\partial x} = \frac{C_x e N_e}{4\pi\epsilon_0} \frac{x}{\mathcal{D}^2} \dot{f}_t \tag{14}$$

$$E^{Cy}_y(t) = \frac{-\partial A^0_{Cx}}{\partial y} = \frac{C_x e N_e}{4\pi\epsilon_0} \frac{y}{\mathcal{D}^2} \dot{f}_t \tag{15}$$

$$E^{Cx}_z(t) = \frac{-\partial A^0_{Cx}}{\partial z} - \frac{\partial A^z_{Cx}}{\partial ct}$$

$$= \frac{C_x e N_e}{4\pi\epsilon_0} \frac{t}{\mathcal{D}^2} \dot{f}_t \tag{16}$$

(17)

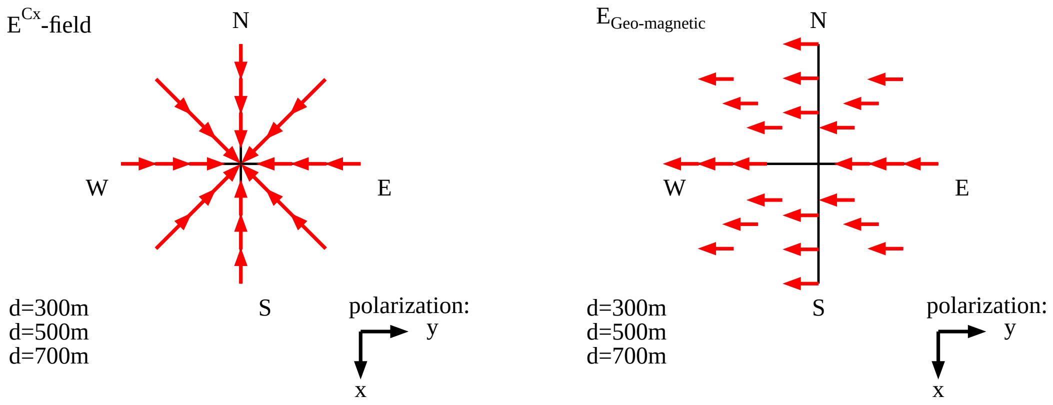
In Fig. 4 the polarization of the electric field is given for different observer positions for both the charge excess component as well as the geo-magnetic radiation. It can be seen that the polarization of the electric field has an observer position dependency when the contribution from the charge excess component is taken into account. Although the magnitude of the total electric field due to charge excess is small compared to the total geomagnetic field, the two contributions can be distinguished by their contributions to the different polarization directions.

III. Conclusions

In a macroscopic picture a simple model can be developed to give insight in the basic features of the radio pulse coming from an air shower. As is shown by making a leading-order expansions at small and large impact parameters, the shape of the electric pulse is determined by different intrinsic quantities of the air shower for observers in the two distance ranges from the shower core. To avoid a divergency coming from the finite number of charged particles hitting the Earth's surface a cut-off parameter a is introduced. It is shown that for impact parameters larger than approximately $d \approx 30$ meters, the pulse shape becomes independent of the cut-off parameter and we can safely use $a = 500$ m for larger impact parameters.

The shape of the electric pulse for small impact parameters is mainly determined by the longitudinal spread of the charged particles within the shower front, whereas for large impact parameters the pulse-shape is determined by the evolution of the total number of particles in the shower over time. This can become an important tool to distinguish air showers coming from different incoming particles.

The inclusion of sub-leading contributions will lead to a polarization of the pulse that is dependent on the observer position. In extreme cases it follows that in a certain polarization direction the only contribution is given by the sub-leading charge excess component of the electric field. From this, important information can be obtained about the Askaryan effect [17], [18]. In this paper, for simplicity, we have restricted ourselves



(a) Charge Excess Field (b) Geo-magnetic Field

Fig. 4: The unit electric field polarization vectors for different observer positions 300, 500, and 700 meters from the impact point. The polarizations are given for a perpendicular incoming air shower with the Earth magnetic field pointing in the $\vec{H} = -H\hat{x}$ direction.

to a simple geometry. We have also calculated several different sub-leading contributions giving approximately a 10% effect. The effects of a realistic index of refraction has also been investigated [14].

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