# Secondary Electron Spectrum from 30 GeV to 10 TeV in the Upper Atmosphere

**Yoshiko Komori***, **Tadashi Kobayashi**†, **Kenji Yoshida**‡ **and Jun Nishimura**§

* *Kanagawa University of Human Services, Yokosuka, Kanagawa 238-8522, Japan*
† *Department of Physics and Mathematics, Aoyama gakuin University, Sagamihara, Japan*
‡ *Department of EIS, Shibaura Institute of Technology, Saitama, Japan*
§ *Institute of Space and Astronautical Science, JAXA, Sagamihara, Japan*

***Abstract*. In the previous paper [1], we have evaluated the secondary electron spectrum in the upper atmosphere using our direct measurements of atmospheric gamma-ray spectrum in the 30 GeV-10 TeV energy range. We have solved the simultaneous equations of electron and gamma-ray spectrum using the cascade shower theory combined by the gamma-ray production rate in the atmosphere. In this paper we present more precise solutions taking into account some nuclear decay modes, particularly, Dalitz decay mode which produces pair electrons directly from neutral pions, contributing about 10% to the secondary spectrum. The analytic results solved exactly are compared with those of the Monte Carlo simulation(MC), and the secondary electron/gamma-ray ratio shows the good agreement with each other. The contribution of Dalitz electrons increases the importance at the higher altitude, and cannot be negligible in the TeV electron balloon measurements. Our observed atmospheric gamma-ray spectrum is 20% higher than the results of MC as indicated previously [2], so that the estimates of secondary electrons become higher than 20% than those of MC. Although the discrepancy still remains, for the correction of primary electron spectrum we use the secondary spectrum obtained from the measured gamma-ray spectrum, because it does not include uncertainties of nuclear interaction models of MC.**

***Keywords*: secondary electrons, nuclear interaction, atmosphere**

## I. Introduction

In balloon-borne electron experiments, both primary electrons coming from interstellar space and secondary electrons produced in the residual atmosphere are observed together. Thus we have to subtract the secondary electrons from the observed electrons for estimating the primary electron spectrum. Secondary electrons above 30 GeV mostly originate in conversion process of atmospheric gamma rays produced in the hadronic interactions in the atmosphere.

The first precise estimates of secondary electron flux is calculated by Orth & Buffington [3]. They give the atmospheric electrons contributed from pions and kaons. In this paper we treat both atmospheric electrons and gamma rays by solving the electromagnetic cascade equation. We succeeded to get the solutions exactly and discuss the behavior both electron and gamma-ray spectrum in the upper atmosphere. The final purpose of this calculation is to estimate the background electrons to derive the primary electron spectrum in balloon experiments. The contribution of secondary electrons is estimated from the observed atmospheric gamma-ray spectrum that is simultaneously observed with electrons for our emulsion chamber experiments.

## II. Calculations

We consider the simultaneous equations of the electron and gamma-ray spectrum above 30 GeV in the atmosphere ($< 10\mathrm{g/cm^2}$). That is constructed by the cascade shower theory of Approximation A by adding the gamma-ray production rate per unit depth. We estimate the rate from the gamma-ray spectrum $J_\gamma$ which is observed simultaneously in our primary electron experiments [2]. The spectrum is given by

$$J_\gamma(E)(\mathrm{m^2 \cdot s \cdot sr \cdot GeV})^{-1} = (1.12 \pm 0.13) \times 10^{-4} (E/100\mathrm{GeV})^{-2.73 \pm 0.06} \quad (1)$$

This spectrum is the vertical gamma-ray spectrum normalized at 4.0 g/cm$^2$, and includes not only gamma rays originated from $\pi^0$ decay mode but from $\eta$ and Kaon decay modes.

### A. Formulation and Solution

The number of electrons $\pi(E,t)dE$ and gamma-rays $\gamma(E,t)dE$ with energies between $E$ and $E+dE$ satisfy the following simultaneous equations (the notations are shown in [5], etc). The cross section agrees with that used in Geant4 simulation code [6], and with MC results as shown in Appendix.

$$\frac{\partial \pi(E,t)}{\partial t} = -A'\pi(E,t) + B'\gamma(E,t) + \pi_{ex}(E,t)$$
$$\frac{\partial \gamma(E,t)}{\partial t} = C'\pi(E,t) - \sigma_0\gamma(E,t) + \gamma_{ex}(E,t) \ ,$$

in which we add the gamma-ray spectrum produced by nuclear interactions.

$$\gamma_{ex}(E,t)dE = k_g E^{-\beta-1} \exp(-t/L) dE$$

We put the attenuation path length $L = 100\ \mathrm{g/cm^2}$. Gamma rays are mostly produced through the $\pi^0$ decay mode, and other small contributions exist, which is estimated by nuclear interaction model of Dpmjet3. The fraction to the $\pi^0$ decay mode has the value of 0.16 and 0.03 for the $\eta$ and Kaon decay mode respectively [2].

A direct production of electrons, namely not reduction from gamma rays, is included in the Dalitz decay mode, which is secondly major mode of $\pi^0$ decay and produce one real gamma ray and one electron-positron pair, and occupy 1.2% of $\pi^0$ decay process. We put the portion of Dalitz electron pair as the electron production rate,

$$\pi_{ex}(E,t)dE = a \cdot \gamma_{ex}(E,t)\ .$$

The coefficient $a$ is estimated from the product of two ratios. The first ratio is the gamma rays from $\pi^0$ to the total gamma rays from nuclear interaction, which is estimated to be $1/(1+0.16+0.03) = 0.84$. The second ratio is the branching ratio of Dalitz electrons to the gamma rays from $\pi^0$. Thus the value of coefficient $a = 5.0 \times 10^{-3}$ is obtained from the product of both ratios.

The initial values, namely primary electron spectrum [4] and the Galactic diffuse gamma-ray spectrum are assumed as

$$\begin{aligned}\pi(E,0) &= 1.6\times10^{-4}\left(\frac{E}{100\ \mathrm{GeV}}\right)^{-3.3}\\ &\quad (\mathrm{m^2\cdot s\cdot sr\cdot GeV})^{-1}\\ \gamma(E,0) &= 0\ .\end{aligned}$$

The above expressions give the index $\alpha = 2.3$ and $\beta = 1.73$ and the simultaneous equation is solved exactly.

The solution of electron intensity at the depth $t$ becomes

$$\pi(E,t) = \pi(E,0)\zeta(t) + \gamma_{ex}(E)\xi(t) \tag{2}$$

$$\begin{aligned}\zeta(t) &= \frac{[(\lambda_1+\sigma_0)e^{\lambda_1 t} - (\lambda_2+\sigma_0)e^{\lambda_2 t}]}{\lambda_1-\lambda_2}\\ \xi(t) &= \frac{B(\beta)}{\lambda_1-\lambda_2}\left[\frac{e^{(\lambda_1+\frac{1}{L})t}-1}{\lambda_1+1/L} - \frac{e^{(\lambda_2+\frac{1}{L})t}-1}{\lambda_2+1/L}\right]\\ &\quad + \frac{a}{\lambda_1-\lambda_2}\times\\ &\quad \left[\frac{(\lambda_1+\sigma_0)e^{(\lambda_1+1/L)t}-(\sigma_0-1/L)}{\lambda_1+1/L}\right.\\ &\quad \left.-\frac{(\lambda_2+\sigma_0)e^{(\lambda_2+1/L)t}-(\sigma_0-1/L)}{\lambda_2+1/L}\right]\end{aligned}$$

where $\zeta(t)$ represents the energy loss rate of electrons by bremsstrahrung in the atmosphere and the first term of $\xi(t)$ represents the pair production rate from gamma rays and the second term is the electron production rate from Dalitz decay mode.

The atmospheric gamma-ray spectrum at depth $t$ is given by

$$\gamma(E,t) = \pi(E,0)\eta_1(t) + \gamma_{ex}(E)\eta_2(t) \tag{3}$$

$$\begin{aligned}\eta_1(t) &= \frac{C(\alpha)}{\lambda_1-\lambda_2}[e^{\lambda_1 t} - e^{\lambda_2 t}]\\ \eta_2(t) &= \frac{1}{\lambda_1-\lambda_2}[f(\lambda_1) - f(\lambda_2)]\\ f(\lambda) &= \frac{1}{\lambda+1/L}\{(\lambda + A(\beta))e^{(\lambda+1/L)t}\\ &\quad -(A(\beta)-\frac{1}{L}) + aC(\beta)(e^{(\lambda+1/L)t}-1)\}\end{aligned}$$

where $\eta_1$ represents the gamma-ray production rate from primary electrons and $\eta_2$ represents the attenuation rate of gamma rays by pair creation.

The coefficients, $\lambda_1$, $\lambda_2$, A(s), B(s) and C(s) are shown in the previous paper [1]. The coefficients $\lambda_1$, $\lambda_2$ are given by The approximate expression of secondary electron spectrum becomes

$$\xi(t)\cdot\gamma_{ex}(E,t) \sim \left(\frac{B(\beta)}{2}t^2 + a\cdot t\right)\gamma_{ex}(E,t)\ ,$$

which corresponds to the term, $x_0^2/\lambda_\gamma + 0.0117x_0$, in the Orth&Buffington's paper [3]

The production rate of gamma rays estimated from the eq (3) at 4.0 g/cm$^2$ using the eq (1) are given by

$$\begin{aligned}\gamma_{ex}(E) &= 1.1\times10^{-3}(E/100)^{-2.73}\exp\left(\frac{-t}{100\mathrm{g/cm^2}}\right)\\ &\quad (\mathrm{m^2\cdot s\cdot sr\cdot GeV\cdot c.u.})^{-1}\ .\end{aligned}$$

### *B. Solution with Zenith Angle Distribution*

As cosmic rays come in all directions, the intensities in the previous section should be integrated with the zenith angle to the threshold $\theta$. At the depth $t$ r.l. , the electron differential spectrum $j_{ob}(E,t)$ and the atmospheric gamma-ray spectrum $\gamma_{ob}(E,t)$ is actually given as follows.

$$\begin{aligned}& j_{ob}(E,t)/2\pi\quad (\mathrm{m^2\cdot sec\cdot GeV^{-1}})\\ &\quad = \int_0^\theta \pi(E,\frac{t}{\cos\theta})\cos\theta\sin\theta d\theta\\ &\quad = \pi(E,0)\zeta(\alpha,\theta,t) + \gamma_{ex}(E)\xi(\beta,\theta,t)\end{aligned}$$

$$\begin{aligned}& \gamma_{ob}(E,t)/2\pi\quad (\mathrm{m^2\cdot sec\cdot GeV^{-1}})\\ &\quad = \int_0^\theta \gamma(E,\frac{t}{\cos\theta})\cos\theta\sin\theta d\theta\\ &\quad = \pi(E,0)\eta_1(\alpha,\theta,t) + \gamma_{ex}(E)\eta_2(\beta,\theta,t)\end{aligned}$$

The coefficients are calculated in the series.

$$\begin{aligned}\zeta(\alpha,\theta,t) &= O_1(\alpha,\theta,t) + \sigma_0 O_2(\alpha,\theta,t)\\ \xi(\beta,\theta,t) &= B(\beta)O_3(\beta,\theta,t)\\ &\quad + a\cdot\{O_2(\beta,\theta,t) + \sigma_0 O_3(\beta,\theta,t)\}\\ \eta_1(\alpha,\theta,t) &= C(\alpha)O_2(\alpha,\theta,t)\\ \eta_2(\beta,\theta,t) &= O_2(\beta,\theta,t)\\ &\quad + (A(\beta) + aC(\beta))O_3(\beta,\theta,t)\end{aligned}$$

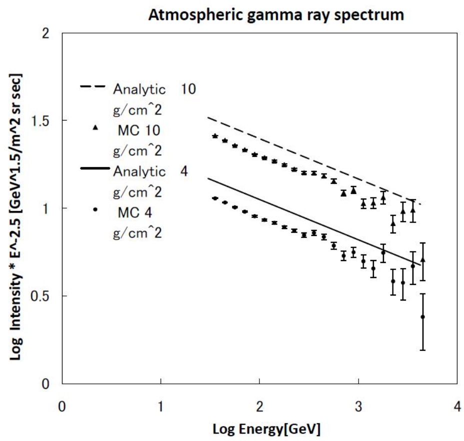


Fig. 1: Gamma-ray spectrum calculated from eq. (3) using eq. (1) and compared with Monte Carlo simulation

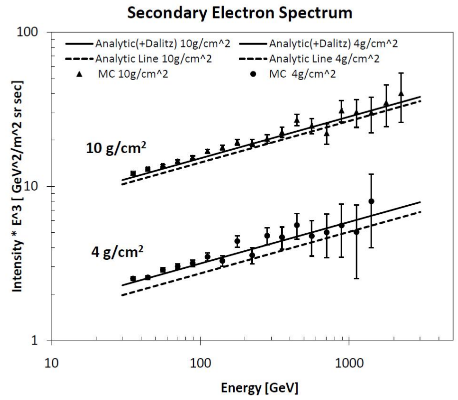


Fig. 2: Secondary electron spectrum. Analytic solution from gamma-ray conversion, and added Dalitz decay mode are shown with MC results divided by 0.80.

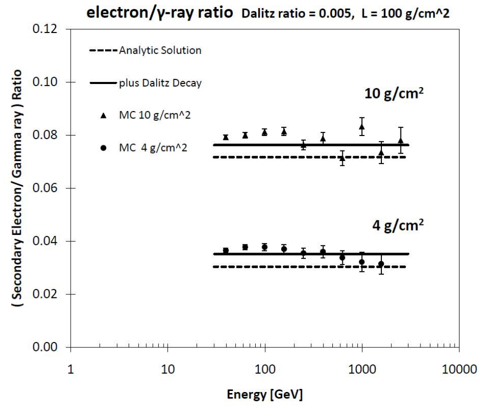


Fig. 3: Secondary electron / gamma-ray ratio calculated analytically and compared with Monte Carlo simulation

and the series are given by

$$O_l(s,\theta,t) = 2\pi \sum_{n=0}^{\infty} \Lambda_n(s) T_{n-1+l}(\theta,t)$$

$$T_0 = \frac{1-\cos^2\theta}{2}, \quad T_1 = t(1-\cos\theta),$$

$$T_2 = \frac{t^2}{2!}\log(\frac{1}{\cos\theta}), \quad T_3 = \frac{t^3}{3!}(\frac{1}{\cos\theta}-1),$$

$$\cdots \quad T_n = \frac{t^n}{n!}(\frac{1}{\cos^{n-2}\theta}-1) \quad \cdots$$

$$\Lambda_0 = \frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_2}, \cdots, \Lambda_n = \frac{\lambda_1^{n+1}-\lambda_2^{n+1}}{\lambda_1-\lambda_2} + \Lambda_{n-1}\frac{-1}{L}, \cdots$$

## III. RESULTS

The analytic solutions are compared with the Monte Carlo simulation, in which we adopted the Dpmjet3 [7] hadronic interaction model, the Cosmos [8] propagation model, and the BESS-TeV [9][10] and RUNJOB [11] data for the input proton, helium, and CNO spectrum at the top of atmosphere.

We show the results of gamma-ray and secondary electron spectrum, the ratio of both spectrum, and the correction of secondary electrons to the observed data.

### A. Atmospheric Gamma-ray Spectrum

The solution of atmospheric gamma rays are given by eq. (3) and shown in Fig. 1 with MC result at the depth of 4, 10 $\mathrm{g/cm^2}$. The solution is 20% higher than the MC result, that is consistent with our previous result [2], in which we have reported the proton spectrum deconvolved from our observed gamma-ray spectrum using the Dpmjet3 model and compared the spectrum with the primary proton data of BESS, etc. The result has shown that our proton spectrum is 20% higher than the primary data. The uncertainties in deriving the flux of primary protons from atmospheric gamma-ray flux come mainly from the hadronic interaction models [2].

### B. Secondary Electron Spectrum

The solution of the secondary electron spectrum at the depth $t$ is given by $j_{sec}(E,t) = 2\pi\xi(t)\gamma_{ex}(E)$ as shown in eq. (2). The comparison with the MC is shown in Fig. 2. We use the MC results divided by 0.80 to correct the discrepancy of gamma-ray spectrum. We notice that Dalitz decay mode contributes at the small but evaluated rate in the high altitude above 4 $\mathrm{g/cm^2}$.

The secondary electrons contribute more largely in the higher energy region because of steeper spectrum of primary electrons than the gamma-ray spectrum. The intensity is proportional to the square of the balloon floating altitude.

### C. Secondary Electron to Gamma-Ray Ratio

It is important to consider the ratio of Secondary electrons to gamma rays because the ratio only concerns the production process of secondary electrons. We know the Dalitz decay mode contributes the constant value to the ratio. It means that the Dalitz mode gives important role to the higher altitude.

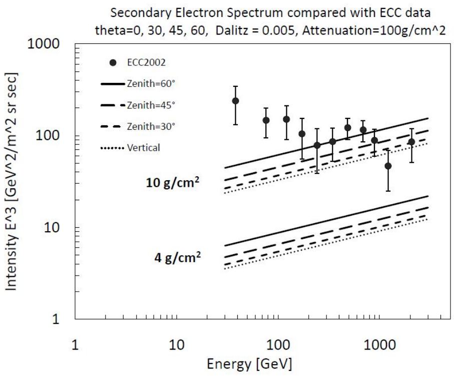


Fig. 4: Secondary electron spectra for correction (the second term in eq.(4)) with different zenith angle distributions, compared with the observed data [12].

### D. Observation and Correction

The primary electron spectrum is obtained from

$$\pi(E,0) = \frac{j_{ob}(E,t)}{2\pi\zeta} - \gamma_{ex}(E)\frac{\xi}{\zeta} \quad (4)$$

The second term of the right side is the secondary electron contribution from the balloon experiment(the depth $t$, zenith angle $\theta$).

$$\hat{j}_{\rm sec}(E,t)|_\theta \equiv \gamma_{ex}(E)\frac{\xi(\beta,\theta,t)}{\zeta(\alpha,\theta,t)} \quad ({\rm m}^2\cdot{\rm s}\cdot{\rm sr}\cdot{\rm GeV})^{-1} \quad (5)$$

The parameter $\zeta$ represents the energy reduction by bremsstrahrung. The parameter $\xi/\zeta$ is proportional to $t^2$.

All the observed electrons are firstly corrected by bremsstrahrung loss energy and next, statistically subtracted secondary electrons, which is proportional to the mean square of altitude.

Fig. 4 shows the eq. (5) with various $\theta$ and and we know that the electron balloon experiment above 1TeV requires the average altitude above 6 $\mathrm{g/cm^2}$.

## IV. Summary

The secondary electrons are almost produced by atmospheric gamma-ray conversion, and around 10% are produced by Dalitz decay mode directly produced from neutral mesons. We treat both processes analytically and get exact solutions. The solutions are compared with the Monte Carlo simulation, which shows a good agreement by addition of Dalitz Decay mode, though the discrepancy of 20% in the atmospheric gamma-ray intensity exists. Especially the ratio of secondary electrons to gamma rays shows a good agreement between analytic solution and MC value. Dalitz mode shows a constant value in the ratio, that is proportional to the depth, on the contrary to electron pairs from gamma rays proportional to the square of the depth. That means Dalitz mode becomes important at the higher altitude. As the secondary electrons are inevitably included in the observed electrons of the balloon experiments, we have to exactly estimate the number as possible.

In TeV energy region the secondary spectrum becomes dominant at depth $\sim 8\ \mathrm{g/cm^2}$. Thus the electron balloon experiments in the TeV region require the high altitude and the long duration. These conditions are rather severe, but the TeV experiments are necessary for the search of cosmic ray origins.

## Appendix

### A. Calculation of Fundamental Processes

We have treated fundamental processes to check the cross section between analytic solution shown below and Monte Carlo simulation(MC) and get the the excellent agreement within 0.5%. When gamma rays enter the thickness $t$ of the air, the produced number of electrons $\pi(E,t)$dE and gamma rays $\gamma(E,t)$ with energies between E and E+dE satisfy the simultaneous equations,

$$\begin{aligned}\frac{\partial\pi(E,t)}{\partial t} &= -A'\pi(E,t) + B'\gamma(E,t)\\ \frac{\partial\gamma(E,t)}{\partial t} &= C'\pi(E,t) - \sigma_0\gamma(E,t)\end{aligned}$$

If the input spectrum of gamma-ray spectrum is given by

$$\begin{aligned}\pi(E,0) &= 0\\ \gamma(E,0) &= k_g\left(\frac{E}{100\ \mathrm{GeV}}\right)^{-\beta-1},\end{aligned}$$

The solution at depth $t$ becomes

$$\begin{aligned}\pi(E,t) &= \frac{\gamma(E,0)}{\lambda_1-\lambda_2}B(s)[e^{\lambda_1 t} - e^{\lambda_2 t}]|_{s=\beta-1}\\ \gamma(E,t) &= \frac{\gamma(E,0)}{\lambda_1-\lambda_2}[A(s)(e^{\lambda_1 t} - e^{\lambda_2 t})\\ &\qquad +\lambda_1 e^{\lambda_1 t} - \lambda_2 e^{\lambda_2 t}]|_{s=\beta-1}\end{aligned}$$

The electron pair-production spectrum is given by

$$\begin{aligned}\pi(E,t) &= k_e(E/100\ \mathrm{GeV})^{-2.73}\\ k_e &= 6.976\ \ (analytic) \qquad arbitraly\ unit\\ &= 7.00\pm0.04\ \ (MC) \qquad at\ t=0.4\mathrm{g/cm^2}\end{aligned}$$

and $k_e = 6.268(analytic) = 6.28{\pm}0.04(MC)$(arbitraly unit) at $4\mathrm{g/cm^2}$. Both values show excellent agreement.

## References


[1] Y. Komori et al., Proc. of 30th ICRC, 2, 11, 2007
[2] K Yoshida, R Ohmori, T. Kobayashi, Y. Komori, Y. Sato and J. Nishimura, Phys. Rev. D 74, 083511, 2006
[3] C.D. Orth and A. Buffington, Astrophys. J, 206, 312, 1976
[4] J. Nishimura, et al., Astrophys. J., 238, 394, 1980
[5] J. Nishimura, Handbuch der Physik, 46, II, 1, Springer, 1967
[6] Geant4, http://wwwasd.web.cern.ch/wwwasd/geant4/G4UsersDocuments/UsersGuides/PhysicsReferenceManual/html/node27.html
[7] S. Roesler et al., Report NO. SLCA-PUB-8740, Proc. of Monte Carlo 2000,72,23, 2000
[8] K. Kasahara Epics & Cosmos http://cosmos.n.kanagawa-u.ac.jp/
[9] T. Sanuki, M. Motoki et al., Astrophys.J., 545, 1135, 2000
[10] S. Haino, T. Sanuki et al., Phys. Lett. B, 594, 35, 2004
[11] A.V. Apanasenko et al., Astropart. Phys., 16, 13, 2001
[12] T. Kobayashi, Y. Komori, K Yoshida et al., ISAS Research Report, 44, 99, 2002