Postshock turbulence and high energy filaments in young supernova remnants

Alexandre Marcowith *, **Fabien Casse** †

* *Laboratoire de Physique Théorique et d'Astroparticules (LPTA) UMR 5207 CNRS*
Université Montpellier II 13 place E. Bataillon F-34095 Montpellier Cedex 5, France
† *AstroParticule & Cosmologie (APC) UMR 7164 CNRS*
Université Paris Diderot 10, rue Alice Domon et Léonie Duquet F-75205 Paris Cedex 13, France

***Abstract*. The paper investigates the effect of post-shock turbulence evolution over the diffusive shock acceleration (DSA) process for a sample of five young supernova remnants (SNRs). Several mechanisms are considered: resonant and non resonant streaming instabilites upstream, stochastic reacceleration by resonant waves and magnetic fluctuations relaxation or amplification downstream, compression of turbulent scales at the shock front. In the case the turbulence is simply advected downstream, we confirm the result of Parizot et al (2006) that the maximum CRs energies should not go well beyond PeV energies in young SNRs where X-ray filaments are observed. Electron reacceleration by resonant modes downstream should be limited in order to alleviate hard X-ray and radio spectra and large X-ray filaments that are not observed especially for the young SNRs Cassiopeia A, Tycho and Kepler. In case of magnetic field evolution downstream the shock front we provide first analytical estimations of intervals of magnetic field strengths at the shock front that are consistent with the assumption of turbulently limited filaments. Except possibly to the case of Tycho the X-ray filaments are likely limited by the radiative losses unless the magnetic field being of the order or less than $200-300$ μGauss downstream the shock front and the diffusion coefficient limited to its Bohm limit. Conditions for an efficient DSA in fast SNR shocks where the turbulence is relaxing downstream is derived: the turbulence spectrum index β and the energy dependence index of the relaxation scale $\delta_{\rm d}$ have to verify the relation $2-\beta-\delta_{\rm d} \geq 0$. This scaling implies that the non-linear Kolmogorov damping regime is marginally or not efficient enough. Other processes considered in Pohl et al (2005) like Alfvén and fast magnetosonic cascades which produce an exponential relaxation are found to be possible. These results are confirmed using numerical simulations.**

***Keywords*: Particle acceleration - Cosmic-rays**

I. Introduction

Recent high-angular resolution X-ray observations of young supernova remnants (SNRs) like Cassiopeia A, Kepler or Tycho, have revealed the presence of very thin filaments probably associated with the forward shock of the supernova (SN) expanding into the interstellar medium (ISM). These filaments are likely explained by the synchrotron emission of electrons with energies of few tens of TeV ([1], [2] and the references therein). The filament size imposes a lower limit on the magnetic field downstream the shock front that can reach two orders of magnitude above the standard ISM values; e.g. [3]. Close to the maximum electron energies, advective and diffusive transports contribute similarly to the filament width [4]. The previous constraints favor a spatial diffusion coefficient downstream a few times above the Bohm limit and support the standard scenario of diffusive shock acceleration (DSA) in SNRs with a strong magnetic field amplification in the shock precursor. However, the Chandra observations have been obtained in a limited frequency range and diffusion regimes that differ from the Bohm diffusion cannot be excluded based on these observations [5]. Other diffusion regimes modify the particle transport at the highest energies and alterate the way the synchrotron spectrum cut-off is reconstructed from the extrapolation of the radio spectrum [6].

The origin of the magnetic fluctuations is still widely debated. One possibility is that the magnetic field is generated by the relativistic particles themselves through their streaming motion ahead the shock front [7]. Recently, [8] discussed a non-resonant regime of the streaming instability that can generate a very strong turbulent magnetic field (and boost the CR acceleration process) readily at the very early stage of the free expansion phase. [9] did shown that both resonant and non-resonant regimes of the streaming instability have to be considered simultaneously in order to fix the magnetic field strength at the shock front. In fast shocks, the non-resonant instability dominates the magnetic field generation; the level of fluctuation at the shock is found to be similar to the value derived from [8], while the resonant instability dominates in slower shocks. Downstream, the turbulence generated upstream may well relax downstream producing a limitation of the spatial extension of the particle journey [11]. This possibility as not yet been completely investigated in the view of the efficiency of DSA and the maximum energy achievable by the relativistic particles.

This issue is examined in this paper. In section II several effects connected with the particle acceleration are discussed: the compression of the coherence length, particle re-acceleration through second order Fermi acceleration. Section III examines the effects of turbulence relaxation over the high energy filaments and the diffusive shock acceleration efficiency. Especially upper limits on the downstream magnetic field are derived for the relaxation process to control the filaments width. We conclude by a comparison between the X- and gamma-ray filaments produced by synchrotron and Inverse Compton radiation on the cosmic microwave photons respectively.

II. Diffusive shock acceleration in advected turbulence

In this section using analytical and numerical calculations, we consider different effects controlling the transport of electrons around a shock wave where strong magnetic field amplification is occuring. Only the advection of the magnetic field downstream is considered in this section; the case of field relaxation is treated in section III. Several articles treat different aspects of the magnetic field amplification in the SNR shock precursor. This work only considers the streaming instability provoked by the superalfvenic motion of relativistic particles ahead the forward SNR shock. The resonant instability involves wave-particle resonant interaction and produces waves at scales of the particle gyroradius $r_{\rm L}$ [10], [7]. The non-resonant regime has been adapted to the SNR shock waves only recently by [8]; see also [9], [12] for further details. The non-resonant waves are produced, at least in the linear growth phase of the instability at very small scales $\ll r_{\rm L}$. There is still some debate about the possibility for the instability to deeply enter into the non-linear regime and saturate at a magnetic field level $\delta B \gg B_\infty$ [13], [14], [15]. In this work it is accepted that in fast shocks in young SNR that the non-resonant instability dominates the production of magnetic field fluctuations. However, the resonant regime cannot however be disgarded and contribute to the magnetic field fluctuations up to a similar level [9].

This section examines the DSA in case of an efficient turbulence amplification mechanism produces a high magnetic field at the shock precursor. We consider the calculations produced in [1] again where the width of X-ray filaments is used to constrain the magnetic field downstream the shock front. The maximum electron energy is fixed in this model by a balance between particle acceleration and radiative losses. This work invokes the supplementary effect of turbulent scale compression at the shock front. We also address the usually overlooked aspect of stochastic particle acceleration in the downstream flow. All these considerations are tested using an algorithm coupling the equations of hydrodynamics and a set of stochastic differentail equations (SDEs) see [16] [17].

It appears that the inclusion of the turbulence compression at the shock does not modify the conclusions of [1] (see in table I the values of the magnetic field and the ratio of the estimated synchrotron cut-off to the cut-off extrapolated from radio observations). The magnetic field strength downstream is only a lower limit as the X-ray filaments width may well be smaller than the Chandra point spread function. However we further argue here that the magnetic field cannot be much higher than a field corresponding to a fraction of the equipartition between the magnetic energy density and the gas kinetic energy donwtream; in that case the particle reacceleration by the particle interaction with resonant waves is important. The electron reacceleration has several effects: the X-ray filaments become larger than their actual size derived from Chandra observations; the synchrotron spectrum at the shock front is harder than the standard shock acceleration. If the last effect can be confused with the effect of CR shock modification the first effect is not observed. The observed size of the X-ray filaments can give a maximum value of the magnetic field downstream (see table I).

SNR \ Synchrotron	$B_{\rm d}(\mu G)$	$E_{\gamma,cut}/E_{\gamma,obs}$	$B_{\rm d}(mG)$ (FII)
Cas A	558	0.2	2.7
Kepler	433	0.3	2.3
Tycho	586	0.7	1.5
SN 1006	170	0.07	0.56
G347.3-0.5	131	0.05	2.1

TABLE I: Inferred values of the downstream magnetic field and synchrotron photon cut-off energy in the case of a synchrotron dominated rim in the context of a Bohm diffusion regime ($\beta = 1$). The above values have been calculated for $r_{\rm tot} = r_{\rm sub} = 4$ (respectively the total and the subshock compression ratio). The strengths of the limit magnetic field (FII) under which the regular Fermi process to overtake the stochastic Fermi process are displays for each objects in the last column. The surrounding ISM densities are given as approximate and averaged values as follows (in ${\rm cm}^{-3}$ units): Cas A: $n_\infty = 1$ [18], Kepler: $n_\infty = 0.7$ [20], Tycho: $n_\infty = 0.4$ [21], SN1006: $n_\infty = 0.05$ (SE rims [19]), G347.3-0.5: $n_\infty = 1$ (poorly constrained see [22]).

III. Diffusive shock acceleration in relaxed turbulence

We now consider the case where the magnetic field fluctuations can vary over a lengthscale downstream much shorter than a SNR shock radius $R_{\rm SN}$. This scale $\ell_{\rm d}$ can depend on the wave number k of the fluctuations. We will accept in this work a power-law dependence of the relaxation length with the particle energy E in resonnance with the wave number k; i.e. $\ell_{\rm d} \propto E^{\delta_d}$. In this work we consider two main relaxation processes as exposed by [11]; the non-linear Kolmogorov damping and the Alfvén and fast magnetosonic cascading process. The former leads to a spatial power-law magnetic profile and to $\delta_{\rm d} = (3-\beta)/2$ (β is the 1D index of the turbulence spectrum at the shock front), the latter produces an exponential drop of the magnetic field at the shock front and $\delta_{\rm d} = 1/2$ ([11], Marcowith & Casse 2009 in preparation; hereafter MC09).

The procedure used is the following: for different type of relaxation process the stationary turbulent energy spectrum is calculated from a kinetic equation, then the magnetic field profiles and the spatial and momentum diffusion coefficients are derived downstream. The spatial diffusion coefficient scales as $E^{2-\beta}$ but depends also on $\delta_{\rm d}$ through the level of resonant magnetic fluctuations. Another way to analyse the problem that permits analytical simple estimates is to consider the magnetic field energy density to be scale invariant over a diffusive zone downstream and then to drop off at a distance $\ell_{\rm d}(k)$; i.e. the turbulent energy density follows by an Heaviside profile (MC09). Once the magnetic profiles, the diffusion coefficients, the particle distribution function around the shock is calculated using the SDE scheme described in [16] and [17] (see also MC09).

We have tested the efficiency of the particle acceleration in the case the drop off of the magnetic fluctuations controls the width of the filaments observed by Chandra. Our main findings can be summarised as follows:

1) Particle acceleration can only be efficient if the relation $2-\beta-\delta_{\rm d} < 0$ is verified. In particular, the non-linear Kolmogorov damping is found to be significantly less efficient than the case controlled by the Alfvenic or the magnetosonic cascade; unless $\beta < 1$. This effect is due to the faster drop of the magnetic profile in the case of Kolmogorov damping over the width of the X-ray filament.
2) In case of Bohm diffusion ($\beta = 1$) particle acceleration also requires the normalisation of the diffusion coefficient with respect to the Bohm coefficient to be close to one; i.e. $q = D(E)/D_{\rm Bohm} \simeq 1$; the diffusion should be close to its Bohm limit.
3) The magnetic field downstream the shockfront in the most favourable case of $q = 1$ cannot be much higher than 400 μGauss in SNR like Cassioppiea A, Tycho or Kepler or 200 μ Gauss in SN1006 or G347.3-0.5. Beyond these values the width of the filaments are controlled by the radiative (i.e. synchrotron) losses.

Table II summarises the limit values of the magnetic field downstream in the case the diffusive random walk of relativistic electrons is the main loss process in the postshock region. Beyond these values radiative losses take over and the X-ray filaments are radiatively-limited.

SNR \ Diff.	$\frac{B_{d-diff}}{q(1)=1}$	$\frac{B_{d-diff}}{q(1)=1}$
Cas A	311	394
Kepler	220	298
Tycho	210	333
SN 1006	174	189
G347.3-0.5	164	183
	$\delta_{\rm d} = 0$	$\delta_{\rm d} = 1/2$

TABLE II: Table presenting analytical estimates of maximum downstream magnetic field in the context of *diffusive loss dominated* SNR rims dor two different values of $\delta_{\rm d}$. The SNR rim observed parameters are the same as in [1] and the shock compression ratio are $r_{\rm tot} = r_{\rm sub} = 4$.

Next, figures 1 and 2 summarise our results concerning the shock particle distribution and magnetic profiles and the X- and gamma-ray filaments expected in case of relaxation of the magnetic field downstream. The calculation have been performed for SNR parameters corresponding to the Kepler SNR (see [1] for further details). We is also seen is that if the magnetic field relaxation controls the particle transport than the maximum particle (electron/proton) energy is highly limited to values close to a few tens of TeV.

The gamma-ray emission produced by all photon energies integrated in two characteristic wavebands 10-30 GeV and 1-3 TeV have been calculated using the standard expression of the isotropic Inverse Compton emissivity [23]. The rims are produced by the scattering off the cosmic microwave photons. Also displayed two wavebands in X-rays; between 4 and 6 keV and 0.5 and 1 keV (even if this band is usually dominated by the thermal component). In each case both projected and deprojected filaments are reproduced.

The relative normalisation between X-ray and gamma-ray filaments mostly depends on the intensity of the magnetic field; it is found to scale as B^2 as expected for the same particle energy domain. The width of the gamma-ray TeV rim is usually the largest as large part of the radiation is produced upstream. The 10-30 GeV gamma-rays are produced closer to the shock upstream with respect to 1-3 TeV gamma-rays. Downstream, the highest energies are confined closer to the shock because of a shorter radiative loss timescale. The projected rims show that only a slight difference exists between the position of the peak of the gamma and X-ray emission. As the size of the gamma-ray rims is not much larger than the X-ray filaments, it is impossible for any actual gamma-ray instrument to separate the two components. This will be also the case for future instruments like CTA unless the filaments being very large (see the case of Vela Junior discussed in [24]).

REFERENCES

[1] E. Parizot, A. Marcowith, J. Ballet, & Y.A. Gallant, 2006, A&A, 453, 387
[2] E.G. Berezhko, 2008, Adv.Sp.Res., 41, 429

Fig. 1: Upper left: Shock Electron distribution multiplied by E^4 in case of an exponential relaxation profile as produced in an Alfvenic or a magnetosonic cascade. The slight modification compared to the linear solution is due to the reacceleration process downstream, the particle index calculated by [5] is shown depending on the downstream alfvenic Mach number $M_{\rm A,d} = V_{\rm d}/V_{\rm A,d}$. Upper right: numerical acceleration timescale versus the particle energy. The downstream residence time is also displayed. Lower left: Spatial dependence of the diffusion coefficient downstream at the maximum electron energy normalised to the coefficient at the shock front. Lower right: Magnetic field profile in case of exponential relaxation. The simulation has been performed for SNR parameters consistent with the data availbale for the Kepler SNR: shock velocity $V_{\rm sh} = 5400$ km/s, upstream mean density $n_\infty = 0.7~{\rm cm}^{-3}$, and a SNR age of 400 years.

Fig. 2: X- and gamma-ray filaments in the case of a Kepler-like SNR. On left are displayed the deprojected filaments, on right the projected filaments. In red the gamma-ray filaments produced by Inverse Compton upscattering the comsic microwave background photons are displayed. In blue the X-ray filaments bewteen two different wavelengths are shown.

[3] E.G. Berezhko, L.T. Ksenofontov, & H.J. Völk, 2003a, A&A, 412, 11
[4] J. Vink, 2004, Nucl. Phys. B Proc. Supp., 132, 21
[5] A. Marcowith, M. Lemoine, & G. Pelletier, 2006, A&A, 453, 193
[6] V.N. Zirakashvili, & F.A. Aharonian, 2007, A&A, 465, 695
[7] A.R. Bell, & S. Lucek, 2001, MNRAS, 321, 433
[8] A.R. Bell, 2004, MNRAS, 353, 550
[9] G. Pelletier, M. Lemoine, & A. Marcowith, 2006, A&A, 453, 181
[10] J. Skilling., 1975, MNRAS, 172, 557
[11] M. Pohl, H. Yan, & A. Lazarian, 2005, ApJ, 626, 101
[12] V.N. Zirakashvili & V.S. Ptuskin, 2008, ApJ, 678, 939
[13] B. Reville, S. O'Sullivan, P.Duffy & J.G. Kirk, MNRAS, 2008, 386, 589
[14] J. Niemiec, M. Pohl, T. Stroman & K.-I., Nishikawa, ApJ, 684, 1174
[15] M.A. Riquelme & A. Spitkovsky, 2009, ApJ, 694, 626
[16] F.Casse & A. Marcowith, 2003, A&A, 404, 405
[17] F.Casse & A. Marcowith, 2005, Aph, 23, 31
[18] E.G. Berezhko, G. Pühlhofer, G. & H.J. Völk, 2003b, A&A, 400, 971
[19] F. Acero, J. Ballet, & A. Decourchelle, A&A, 475, 883
[20] F.A. Aharonian, et al, 2008, A&A, 488, 219
[21] J.P. Hughes, J.P., 2000, ApJ, 545, L53
[22] F.A. Aharonian et al. 2006, Nature, 432, 75
[23] G.R. Blumenthal & R.J. Gould, 1970, RMP, 42, 237
[24] A. Bamba, A., R. Yamazaki & J.S. Hiraga, 2005, ApJ, 632, 294