On Irregular Heliolongitudinal Solar Wind Velocity and Consequences for Galactic Cosmic Ray Intensity Variations

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Abstract. **We develop a three dimensional (3-D) model of the 27-day variation of galactic cosmic ray (GCR) intensity with a spatial variation of the solar wind velocity. A consistent, divergence-free interplanetary magnetic field is derived by solving the corresponding Maxwell equations with a variable solar wind speed, which reproduces in situ observed experimental data. We perform model calculations for the GCR intensity using the variable solar wind and the corresponding magnetic field. Results are compatible with neutron monitors experimental data.**

Keywords: **27- day GCR intensity variation, solar wind, IMF divergence, modelling**

I. INTRODUCTION

Theoretical study of different classes of the galactic cosmic rays (GCR) intensity variations generally is implied by Parker's transport [1] equation with the constant solar wind velocity, and for the interplanetary magnetic field (IMF) B satisfying, equation $divB = 0$. To properly model the 27-day variation and Forbush decreases of the GCR intensity, and propagation of solar cosmic rays, the spatial and time dependencies of the solar wind velocity V, and the IMF must be taken into account. However, it is rather complicated problem, because the validity of the Maxwell's equation $divB = 0$ should be kept for the time and spatially dependent solar wind speed. Maxwell's equations for the IMF B have a form [2], [3]:

$$\begin{cases} \frac{\partial B}{\partial t} & = \nabla \times (V \times B) \\ divB & = 0 \end{cases} \quad (1)$$

where B is the IMF strength, V-solar wind velocity, and t-time. The system of scalar equations for the components $(B_r, B_\theta, B_\varphi)$ of the IMF and components $(V_r, V_\theta, V_\varphi)$ of the solar wind speed corresponding to Eqs. 1 can be rewritten in the heliocentric spherical coordinate system, as:

$$\begin{cases} \frac{\partial B_r}{\partial t} & = \frac{\partial}{\partial \theta}[(V_r B_\theta - V_\theta B_r)r sin\theta] \\ & \quad - \frac{\partial}{\partial \varphi}[(V_\varphi B_r - V_r B_\varphi)r] \\ \frac{\partial B_\theta}{\partial t} & = \frac{\partial}{\partial \varphi}(V_\theta B_\varphi - V_\varphi B_\theta) \\ & \quad - \frac{\partial}{\partial r}[(V_r B_\theta - V_\theta B_r)r sin\theta] \\ \frac{\partial B_\varphi}{\partial t} & = \frac{\partial}{\partial r}[(V_\varphi B_r - V_r B_\varphi)r] \\ & \quad - \frac{\partial}{\partial \theta}[V_\theta B_\varphi - V_\varphi B_\theta] \\ 0 & = \frac{1}{r}\frac{\partial}{\partial r}(r^2 B_r) + \frac{1}{sin\theta}\frac{\partial}{\partial \theta}(sin\theta B_\theta) \\ & \quad + \frac{1}{sin\theta}\frac{\partial B_\varphi}{\partial \varphi} \end{cases} \quad (2)$$

To solve Eqs. 2 in general is difficult, but for our purpose, these equations can be simplified for the particular electro-magnetic conditions on the Sun and in the interplanetary space. Our aim in this paper is to compose a model of the 27-day variation of the GCR intensity for the solar wind speed depending on heliolongitude reproducing in situ measurements for one Sun's rotation for the concrete period of 22 November - 18 December 2007.

II. EXPERIMENTAL DATA

The simultaneous enhancements of the quasi periodic changes of the GCR intensity and parameters of solar wind were noticed by [4] for the positive polarity periods of the solar activity minima epochs. It was shown [5], [6], [7] that the heliolongitudinal asymmetry of the solar wind speed is one of the imperative sources of the 27-day variation of the GCR intensity and anisotropy in the minimum epochs of solar activity. In this paper we analyze experimental data of the daily solar wind velocity, GCR intensity from the Kiel neutron monitor and radial B_x, azimuthal B_y and heliolatitudinal B_z components of the IMF for the period of 22 November - 18 December 2007. Fig. 1 shows that the quasi periodic changes with a period of 27 days are established in all parameters except for the B_z component of the IMF. There is not any recognizable relation of the changes of the B_z component (due to its negligible values) with other parameters; also, it is obvious that the contribution of the B_z component in the changes of the magnitude of the IMF is negligible.

As it was shown [5], [6], [7], the heliolongitudinal asymmetry of the solar wind speed is one of the crucial parameters in creation of the 27-day variation of the GCR intensity in the minimum epochs of solar activity.
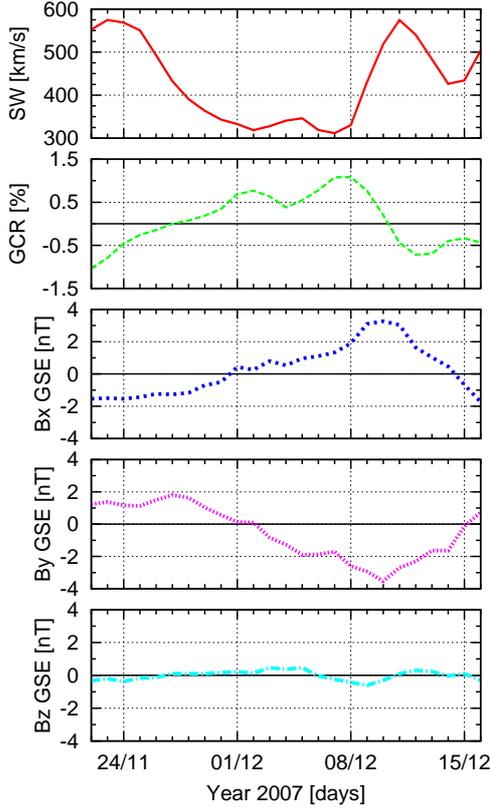
Fig. 1: Temporal changes of the daily (running 3 days smoothing) solar wind velocity (SW) [OMNI], GCR intensity from the Kiel neutron monitor and radial B_x, azimuthal B_y and latitudinal B_z components of the IMF [OMNI] for the period of 22 November - 18 December 2007.

Correlation between the changes of solar wind speed and the GCR intensity is negative and correlation co-efficient equals -0.80 ± 0.05 during the period to be analyzed. The changes of the solar wind velocity, the GCR intensity, B_x, B_y, B_z components of the IMF for one solar rotation period can be considered as a quasi stationary. Value of B_z component of the IMF oscillates near zero in comparison with the changes of the B_x and B_y components so its role could be abandoned in further consideration, as well.

III. MODEL OF THE 27-DAY VARIATION OF THE GCR INTENSITY

Modeling of the 27-day variation of GCR intensity is of interest since [4] found that the recurrent 27-day variation of solar wind parameters as well as that of GCR intensities are $\sim 50\%$ larger for positive $(A > 0)$ polarity epochs of solar magnetic cycles, than for the negative $(A < 0)$. Previously, it was demonstrated [8], [9], [5], [10], [11] that the amplitudes of the 27-day variation of the GCR intensity obtained from neutron monitors are greater in the minimum epochs of solar activity for the $A > 0$ than for the $A < 0$. Recently,

we demonstrated [12], [7], [13], [14] that also the amplitudes of the 27-day variation of the GCR anisotropy at solar minimum are greater when $A > 0$ than when $A < 0$. However, many of the papers [5], [7], [13], [14], [12], [15], [16], [17] aimed to explain results of [4], the general attention was maid to the drift effect and the role of recurrent changes of the solar wind speed, which is a crucial [7], [18] was not considered. In this paper we perform model calculations for the 27-day variation of the GCR intensity using the variable solar wind and the corresponding magnetic field derived from solving the Maxwell equations.

A. Numerical solution of Maxwell's equation

We assume that the changes of the solar wind velocity, the GCR intensity, B_x, B_y and B_z components of the IMF are quasi stationary for one rotation period of the Sun (for instant state of the heliosphere), i.e. the distribution of the GCR density is determined by the time independent parameters. Therefore, we accept $\frac{\partial B_r}{\partial t} = 0$, $\frac{\partial B_\theta}{\partial t} = 0$, $\frac{\partial B_\varphi}{\partial t} = 0$ in Eqs. 2. Also, we accept that average value of the heliolatitudinal component of the solar wind velocity V_θ equals zero; then the system of Eqs. 2 can be reduced, as

$$
\begin{cases}
sin\theta V_r \frac{\partial B_\theta}{\partial \theta} + sin\theta B_\theta \frac{\partial V_r}{\partial \theta} + cos\theta V_r B_\theta \\
-V_\varphi \frac{\partial B_r}{\partial \varphi} - B_r \frac{\partial V_\varphi}{\partial \varphi} + V_r \frac{\partial B_\varphi}{\partial \varphi} + B_\varphi \frac{\partial V_r}{\partial \varphi} = 0 \\
V_\varphi \frac{\partial B_\theta}{\partial \varphi} + B_\theta \frac{\partial V_\varphi}{\partial \varphi} + rsin\theta V_r \frac{\partial B_\theta}{\partial r} \\
+rsin\theta B_\theta \frac{\partial V_r}{\partial r} + sin\theta V_r B_\theta = 0 \\
rB_r \frac{\partial V_\varphi}{\partial r} + rV_\varphi \frac{\partial B_r}{\partial r} + V_\varphi B_r - V_r B_\varphi \\
-rV_r \frac{\partial B_\varphi}{\partial r} - rB_\varphi \frac{\partial V_r}{\partial r} + B_\theta \frac{\partial V_\varphi}{\partial \theta} + V_\varphi \frac{\partial B_\theta}{\partial \theta} = 0 \\
\frac{\partial B_r}{\partial r} + \frac{2}{r}B_r + \frac{ctg\theta}{r}B_\theta + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{1}{rsin\theta}\frac{\partial B_\varphi}{\partial \varphi} = 0
\end{cases}
\quad (3)
$$

The latitudinal component B_θ of the IMF is very feeble for the period to be analyzed, so we can assume that it equals zero $(B_\theta = 0)$, so further in this paper we consider 2D model of the interplanetary magnetic field. This assumption straightforwardly leads (from first equation in system of Eqs. 3) to the relationship between B_r and B_φ, as $B_\varphi = B_r \frac{V_\varphi}{V_r}$, where $V_\varphi = -\Omega r sin\theta$ is the negative corotational speed. Then fourth equation in system of Eqs. 3 with respect to the radial component B_r has a form:

$$
A_1 \frac{\partial B_r}{\partial r} + A_2 \frac{\partial B_r}{\partial \varphi} + A_3 B_r = 0 \quad (4)
$$

The coefficients A_1, A_1 and A_3 depend on the radial V_r and heliolongitudinal V_φ components of the solar wind velocity V. Our goal is to solve Eq. 4 in heliocentric co-ordinate system with a variable solar wind speed, which reproduces in situ measurements in the interplanetary space. In Fig. 2 are presented the daily data of the solar wind speed (points) and dashed curve representing the approximation of the first harmonic wave (27-day variation) during the period of 22 November-18 December 2007. We included in Eq. (4) approximation of the changes of the daily solar wind speed (dashed line in Fig. 2) according to the formula:

$$
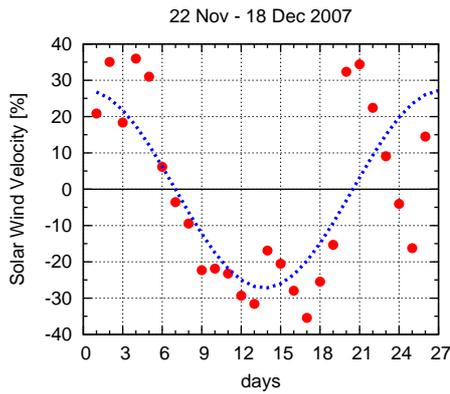V_r = V_0(1 + \alpha sin(\varphi - \varphi_0)) \quad (5)
$$

Fig. 2: Temporal changes of the solar wind speed for one Carrington rotation daily data (points) and dashed curve representing the first harmonic (27 days) wave for the period of 22 November-18 December 2007.

where $\alpha = -0.3$, $\varphi_0 = 1.57$. We take into account, as well

$$V_\theta = 0, V_\varphi = -\Omega r sin\theta \qquad (6)$$

where Ω is the angular velocity of the Sun. Taking into consideration the expressions (5) and (6) the coefficients A_1, A_2 and A_3 in Eq. 4 are $A_1 = 1$, $A_2 = -\frac{\Omega}{V_r}$, $A_3 = \frac{2}{r} + \frac{\Omega}{V_r^2}\frac{\partial V_r}{\partial \varphi}$. Equation 4 is first order linear partial differential equation. It can be solved analytically (e.g. [19]), as well by numerical method. The analytical solution contains arbitrary function which is complicated for our case due to expressions of coefficients A_2 and A_3. So, we solve Eq. 4 by numerical method. Equation 4 was reduced to the algebraic system of equations using a difference scheme method (e.g. [20]), as

$$A_1 \frac{B_r[i+1,j,k] - B_r[i,j,k]}{\Delta r}$$
$$+A_2 \frac{B_r[i,j,k+1] - B_r[i,j,k]}{\Delta \varphi} \quad +A_3 B_r[i,j,k] = 0 \,(7)$$

where, i=1,2,..., I; j=1,2,..., J; k=1,2,..., K are steps in radial distance, vs. heliolatitude and heliolongitude, respectively. Then Eq. 7 was solved by the iteration method with the boundary condition near the Sun $B_r[1,j,k] = const$; in considered case $r_1 = 0.5 AU$ and $B_r[1,j,k] = 25 nT$ for $0^0 < \theta \le 90^0$ and $-25 nT$ for $90^0 < \theta \le 180^0$ for the positive polarity period ($A > 0$). The choice of these boundary conditions was stipulated by requiring agreement of the solutions of Eq. 7 with the in situ measurements of the B_r and B_φ components of the IMF at the Earth orbit. In Fig. 3- 5 are presented results of the solution of Eq. 7 for the B_r and B_φ components of the IMF calculated by the expression $B_\varphi = B_r \frac{V_\varphi}{V_r}$.

B. Modeling of the 27-day variation of the GCR intensity

For modeling the 27-day variation of the GCR intensity we use stationary ($\frac{\partial N}{\partial t} = 0$) Parker's transport
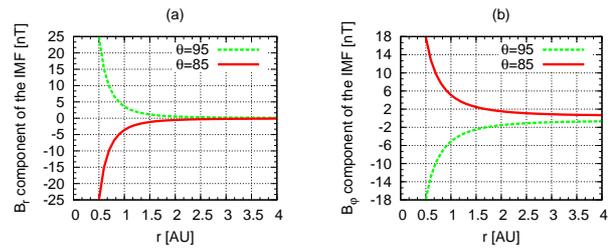

Fig. 3: Radial changes of the (a) B_r and (b) B_φ components of the IMF for different heliolatitudes near the solar equatorial plane for the solar wind speed given by Eq. 5.
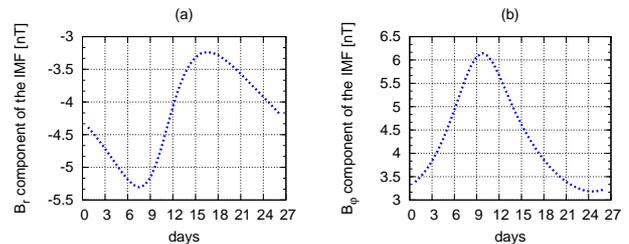

Fig. 4: Azimuthal changes of the (a) B_r and (b) B_φ components of the IMF at the Earth orbit for the solar wind speed given by Eq. 5.
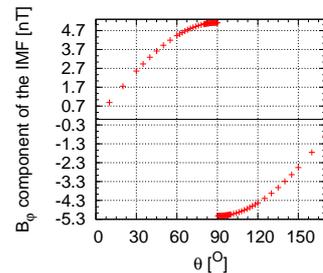

Fig. 5: Heliolatitudinal changes of the B_φ component of the IMF at 1 AU for the solar wind speed given by Eq. 5.

equation [1]:

$$\frac{\partial N}{\partial t} = \nabla_i(K_{ij}\nabla_j N) - \nabla_i(V_i N) + \frac{1}{3}\frac{\partial}{\partial R}(NR)(\nabla_i V_i) \qquad (8)$$

Where N and R are density and rigidity of cosmic ray particles, respectively; V_i - solar wind velocity, t is time, K_{ij} is the anisotropic diffusion tensor of galactic cosmic rays taken from [21]. In this model we assume that the stationary 27-day variation of the GCR intensity is caused by the heliolongitudinal asymmetry of the solar wind speed. In Eq. 8, we included the changes of the solar wind speed (Eq. 5), which reproduces in situ measurements (Fig. 2). In Parker's transport equation we included B_r and B_φ components and the magnitude $B = \sqrt{B_r^2 + B_\varphi^2}$ of the IMF obtained from the numerical solution of Eq. 7 with a variable solar wind speed, as well. Implementation of the heliospheric

magnetic field obtained from the numerical solution of Eq. 7 in Parker's transport equation is done through the spiral angle $\psi = arctan(-\frac{B_\varphi}{B_r})$ in anisotropic diffusion tensor of GCR particles (ψ is the angle between magnetic field lines and radial direction in the equatorial plane).The kinematical model of the IMF with variable solar wind speed has some limitations, especially it would be applied until some radius, while at large radii the faster wind would overtake the previously emitted slower one. To exclude an intersection of the IMF lines the heliolongitudinal asymmetry of the solar wind speed takes place only up to the distance of ~ 8 AU and then $V = 400$ km/s throughout the heliosphere. In connection with this behind 8 AU in the theoretical model of standard Parker's field is used. Equation 8 was solved numerically as in our papers published elsewhere [21], [11], [6], [22]. Changes of the relative density obtained as a solution of the transport Eq. 8 for the model of the 27-day variation of the GCR intensity corresponding to the analyzed period are presented in Fig. 6 (dashed line); in this figure are also presented (points) changes of the GCR intensity obtained by Kiel neutron monitor experimental data for the period of 22 November - 18 December 2007 (Fig. 1), as well. Fig. 6 shows that results of theoretical modeling (dashed
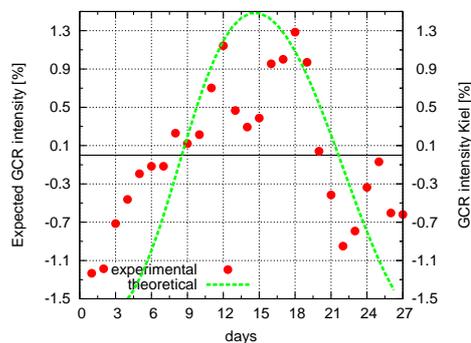

Fig. 6: Heliolongitudinal changes of the expected GCR intensity for rigidity 10GV at the Earth orbit during solar rotation period (dashed line) and temporal changes of the GCR intensity from the Kiel neutron monitor during the period of 22 November - 18 December 2007 (points).

line) and the experimental data (points) are in good agreement. We underline that the presented model of the 27-day variation of the GCR intensity composed for the variable solar wind speed (5) and the IMF's components B_r and B_φ obtained as the solution of Eq. 7 is compatible with the neutron monitors experimental data.

IV. CONCLUSIONS

1) The quasi steady 27-day variations of the solar wind velocity, GCR intensity, B_r and B_φ components of the IMF have been analyzed for the period of 22 November - 18 December 2007.

2) The Maxwell equations are solved with a solar wind speed varying in heliolongitude in accordance with in situ measurements in case to derive the longitudinal dependence of the B_r and B_φ components of the IMF.

3) A three-dimensional model is proposed for the 27-day variation of GCR intensity in response to a realistic variation of the solar wind velocity. The model incorporates the B_r and B_φ components of the IMF derived from solving the Maxwell equations.

4) The proposed model of the 27-day variation of the GCR intensity is in good agreement with the observational material.

V. ACKNOWLEDGMENTS

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