Model of the Forbush decrease of the galactic cosmic ray intensity with the spatial dependent solar wind velocity and comparison with the experimental data

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***Abstract*. We develop three dimensional model of the Forbush decrease (Fd) of the galactic cosmic (GCR) intensity including the changes of the solar wind velocity. We show that the results of the theoretical modelling are in good agreement with the experimental data. Also, we show that the temporal changes of the expected rigidity spectrum exponent γ in different phases of the Fd does not depend on the level of convection of the GCR stream, but depends only on the temporal changes of the interplanetary magnetic field turbulence.**

***Keywords*: Forbush decrease, rigidity spectrum, IMF turbulence, modelling**

I. Introduction

Based on the experimental data analysis we have showed [1]-[8] that the changes of the exponent γ of the rigidity R spectrum $\delta D(R)/D(R) \propto R^{-\gamma}$ of the Forbush decreases (Fd) of the galactic cosmic ray (GCR) intensity found by neutron monitors and ground muon telescopes experimental data is related with the changes of the power spectral density (PSD) of the interplanetary magnetic field (IMF) turbulence ($PSD \propto f^{-\nu}$, f is a frequency); namely the exponent γ depends on the exponent ν in the range of frequency f of the IMF turbulence, $f \sim 10^{-6} Hz \div 10^{-5}$ Hz, to which neutron monitors and ground muon telescopes respond. A relationship between the exponent γ and the exponent ν exists ([1]-[8]) owing to the dependence of the diffusion coefficient K of GCR particles on the rigidity R as $K \propto R^{\alpha}$, i.e. that the exponent α is proportional to the γ. According to the quasi linear theory (QLT), the coefficient α depends on the exponent ν of the PSD of the IMF turbulence, as $\alpha = 2-\nu$ ([9], [10], [11], [12]), so there should exists the relationship between exponent γ of the rigidity spectrum and the exponent ν of the PSD of the IMF turbulence in the form $\gamma \approx 2 - \nu$.
An applying the QLT for the GCR particles with the energy $\geq$ 1 GeV is justified since for particles with these energy all newly presented theories like nonlinear guiding center theory (NLGCT) [13], weakly nonlinear theory (WNLT) [14] and QLT give the same results [15], [16].
Our aim in this paper is to propose the three dimensional stationary model of the Fd taking into account the increase of the solar wind velocity during the Fd and to estimate the influence of the convection of the GCR stream on the rigidity spectrum of the Fd.

II. Experimental Data Analysis

For the analyze we consider the period of 24 August-10 October 2005. In Fig. 1 are presented changes of the B_x, B_y and B_z components of the IMF (from ACE), GCR intensity by Moscow neutron monitor, DST index and solar wind speed. We study the temporal changes of the rigidity spectrum of Fd of the GCR intensity occurred in 9-25 September 2005. To study Fd

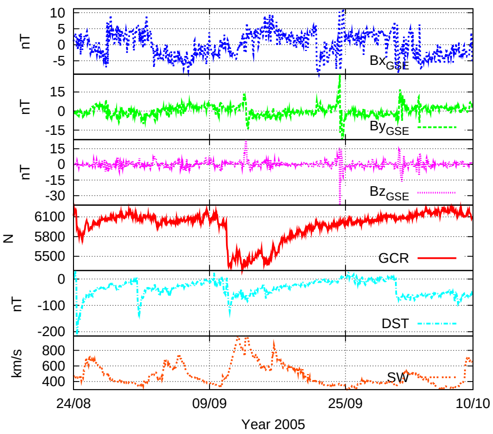

Fig. 1: Changes of the B_x, B_y and B_z components of the IMF (from ACE), GCR intensity by Moscow neutron monitor, DST index and solar wind speed (SW) in the period of 24 August - 10 October 2005.

we use daily average data of the 14 neutron monitors (Apatity, Calgary, Climax, Fort Smith, Halekala, Irkutsk, Jungfraujoch, Lomnicky Stitt, Moscow, Oulu, Tbilisi, Thule, Tixi Bay, Yakutsk) and 6 different channels of Nagoya muon telescope (N0VV, N1WW, N1NN, N4NE, N4NW, N4SE) (upper panel of Fig. 2). The exponent γ of the power law rigidity spectrum was found using the expression e.g.[17]:

$$\frac{\delta D(R)}{D(R)} = \begin{cases} AR^{-\gamma} & \text{for } R \leq R_{max} \\ 0 & \text{for } R > R_{max} \end{cases}$$

Where R_{max} is rigidity beyond which the Fd of the GCR intensity vanishes. The method of calculation is described in papers [1], [3]. Fig. 2 (bottom panel) shows that rigidity spectrum during the beginning and recovery phases of the Fd is relatively soft with respect to the rigidity spectrum in the minimum and near minimum phases of the GCR intensity. The temporal changes of

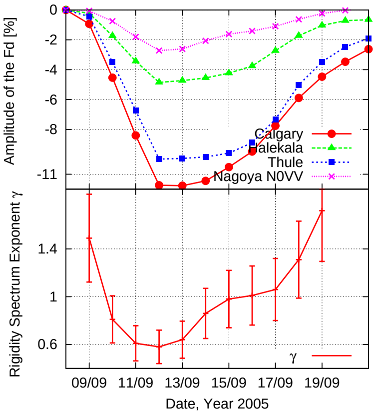

Fig. 2: Temporal changes of the GCR intensity (top panel) for Calgary, Halekala and Thule neutron monitors and Nagoya N0VV muon telescope and of the rigidity spectrum exponent γ (bottom panel) for period of 9-20 September 2005.

the exponent γ (bottom panel of Fig. 2) we ascribe to the conversion of the structure of the IMF turbulence during the Fd. Particularly, the hardening of the rigidity spectrum γ of the Fd (the exponent γ gradually decreases in the minimum and near minimum phases of the Fd) should be observed owing to the increase of the exponent ν of the PSD in the energy range of the IMF turbulence ($f \sim 10^{-6}Hz \div 10^{-5}$ Hz). To estimate a relation between γ and ν there should be found PSD of the IMF components for the time interval long enough for the acceptable range of frequency ($f \sim 10^{-6}Hz{\div}10^{-5}$ Hz). What is worse, the periods before and after the Fd should be relatively quiet to compare with the period of the Fd (period of disturbances). To investigate the evolution of the IMF turbulence during the Fd in September 2005 we consider three periods, first one (I: 24 August - 8 September 2005) - before the Fd, the second one (II: 9-24 September 2005) -during Fd, and the third one (III: 25 September- 10 October 2005) after the Fd. Fig. 3 shows that during the Fd (II period) the exponents ν are greater for B_y and B_z components of the IMF, than before (I period) and after the Fd (III period). So, during the Fd is observed the inverse dependence between the changes of the exponents γ and ν ; when the exponent ν increases the exponent γ decreases.

III. MODELING OF THE FORBUSH DECREASE

To describe the Fd of the GCR we use the Parker's transport equation [18]:

$$\frac{\partial N}{\partial t} = \nabla_i(K_{ij}\nabla_j N) - \nabla_i(V_i N) + \frac{1}{3}\frac{\partial}{\partial R}(NR)(\nabla_i V_i) \tag{1}$$

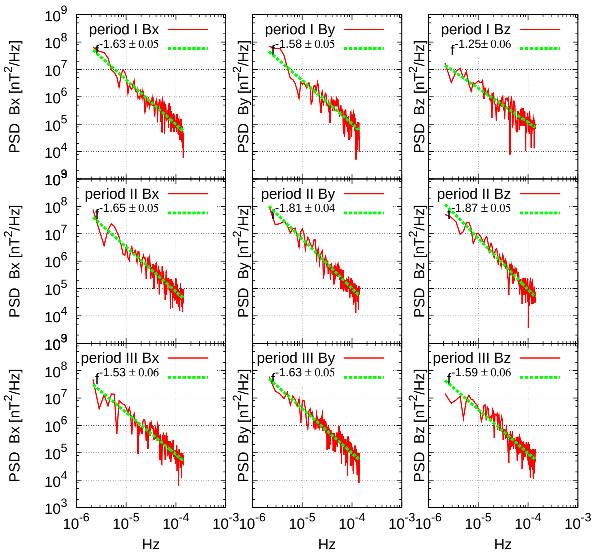

Fig. 3: Power Spectrum Density of the B_x, B_y and B_z components of the IMF for the periods before (I), during (II) and after (III) the Fd in September 2005.

Where N and R are density and rigidity of cosmic ray particles, respectively; U_i - solar wind velocity, K_{ij} - is the anisotropic diffusion tensor of cosmic rays. An applying of the stationary model ($\frac{\partial N}{\partial t} = 0$) to describe the Fd is justified, when the amplitudes of the Fd of GCR intensity generally are $< 5-6\%$ in the energy range of 10 GeV and duration is reasonably large (10-12 days). In the stationary model the Fd of the GCR intensity is realized by the changes of the expected density versus the heliolongitudes (the value of the heliolongitudes 13.3° corresponds to the 1 day). We set up the dimensionless density $f = \frac{N}{N_0}$ and distance $r = \frac{r'}{r_0}$. N and N_0 are density in the interplanetary space and in the local interstellar medium (LISM), respectively; r' is the distance from the Sun and r_0 the region of the modulation. The density N_0 of GCR is $N_0 = 4\pi I_0$, where the intensity I_0 in the LISM [19] has the form $I_0 = \frac{21.1T^{-2.8}}{1+5.85T^{-1.22}+1.18T^{-2.54}}$; T is kinetic $T = \sqrt{R^2+0.938^2} - 0.938$. The Eq. 1 for the dimensionless variables f and r in the spherical coordinate system (r,θ,φ) can be written:

$$A_1\frac{\partial^2 f}{\partial r^2} + A_2\frac{\partial^2 f}{\partial \theta^2} + A_3\frac{\partial^2 f}{\partial \varphi^2} + A_4\frac{\partial^2 f}{\partial r\partial\theta} + A_5\frac{\partial^2 f}{\partial\theta\partial\varphi} + A_6\frac{\partial^2 f}{\partial r\partial\varphi} + A_7\frac{\partial f}{\partial r} + A_8\frac{\partial f}{\partial\theta} + A_9\frac{\partial f}{\partial\varphi} + A_{10}f + A_{11}\frac{\partial f}{\partial R} = 0 \tag{2}$$

The coefficients A_1, A_2,..., A_{11} are functions of the spherical coordinates (r,θ,φ) and rigidity R of GCR particles. To construct a realistic model of the Fd we have to take into account, the change of the solar wind velocity and corresponding three dimensional IMF (B_r, B_θ, and B_φ components) depending on the spatial coordinates; however, in this case the validness of the Maxwell's equation $divB = 0$ should be kept for the time and spatially dependent solar wind velocity. Maxwell's equations for the IMF strength B have a form [20]:

$$\begin{cases} \frac{\partial B}{\partial t} & = \nabla \times (V \times B) \\ divB & = 0 \end{cases} \quad (3)$$

So, we have to solve the system of equation (3) to obtain the IMF components corresponding to the assumed solar wind velocity. To solve the system of Eqs. (3) in general form is difficult, but for our purpose, it can be simplified. We consider a stationary case ($\frac{\partial B_r}{\partial t} = 0$, $\frac{\partial B_\theta}{\partial t} = 0$, $\frac{\partial B_\varphi}{\partial t} = 0$) and assume that an average value of the heliolatitudinal component of the solar wind velocity V_θ equals zero. Then the system of equations (3) in the heliocentric spherical (r, θ, φ) coordinate system can be reduced, as

$$\begin{cases} sin\theta V_r \frac{\partial B_\theta}{\partial \theta} + sin\theta B_\theta \frac{\partial V_r}{\partial \theta} + cos\theta V_r B_\theta \\ -V_\varphi \frac{\partial B_r}{\partial \varphi} - B_r \frac{\partial V_\varphi}{\partial \varphi} + V_r \frac{\partial B_\varphi}{\partial \varphi} + B_\varphi \frac{\partial V_r}{\partial \varphi} = 0 \\ V_\varphi \frac{\partial B_\theta}{\partial \varphi} + B_\theta \frac{\partial V_\varphi}{\partial \varphi} + rsin\theta V_r \frac{\partial B_\theta}{\partial r} \\ +rsin\theta B_\theta \frac{\partial V_r}{\partial r} + sin\theta V_r B_\theta = 0 \\ rB_r \frac{\partial V_\varphi}{\partial r} + rV_\varphi \frac{\partial B_r}{\partial r} + V_\varphi B_r - V_r B_\varphi \\ -rV_r \frac{\partial B_\varphi}{\partial r} - rB_\varphi \frac{\partial V_r}{\partial r} + B_\theta \frac{\partial V_\varphi}{\partial \theta} + V_\varphi \frac{\partial B_\theta}{\partial \theta} = 0 \\ \frac{\partial B_r}{\partial r} + \frac{2}{r} B_r + \frac{ctg\theta}{r} B_\theta + \frac{1}{r} \frac{\partial B_\theta}{\partial \theta} + \frac{1}{rsin\theta} \frac{\partial B_\varphi}{\partial \varphi} = 0 \end{cases} \quad (4)$$

We consider 2D IMF so the latitudinal component B_θ of the IMF equals zero ($B_\theta = 0$). This assumption straightforwardly leads (from first equation in (4)) to the relationship between B_r and B_φ, as $B_\varphi = B_r \frac{V_\varphi}{V_r}$, where V_φ is the negative corotational speed. More details of the solution of the system of equations (4) for two dimensional IMF are described in [21].

Our aim is to show theoretically a dependence of the expected rigidity spectrum exponent γ of the Fd on the exponent ν of the PSD of the IMF turbulence, so in the models we assume that the Fd of the GCR intensity is caused by the change of the diffusion coefficient owing to the changes of the IMF turbulence. In our previous papers [1]-[8] was shown that the temporal changes of the rigidity spectrum of the Fd of the GCR intensity found by neutron monitors experimental data can be provided from the theoretical modeling only if the change of the IMF turbulence is assumed. However, previously we did not considered the change of the solar wind velocity and corresponding changes of the IMF components. In the presented models the diffusion coefficient is $K_\parallel = K_0 K(r) K(R, \varphi)$, where $K_0 = 4.5 \times 10^{21}\ cm^2/s$, $K(r) = 1 + 0.5 \frac{r}{1AU}$. To show the dependence of the exponent γ on the exponent ν, we assume in all models the change of the diffusion coefficient due to the increase of the IMF turbulence in the lower frequency region i.e. we suppose that the exponent ν in the vicinity responsible for the Fd increases as:

$$\nu(\varphi) = 0.8 + 0.5 * (cos(\varphi) + 0.2) \quad (5)$$

At the same time we include in our models various changes of the solar wind velocity by the formula

$$V(\varphi, \theta) = 400 * [1 + \delta * (-exp(-0.7\varphi + 80^\circ) \\ cos(1.1\varphi)] * (0.2 + sin(\theta)) \quad (6)$$

where the maximal velocity is changing by expense of the coefficient $\delta = \{0.25, 0.5, 0.75, 1.25\}$ from 480 to 780 km/s. (Fig. 4). The B_r and B_φ components of the

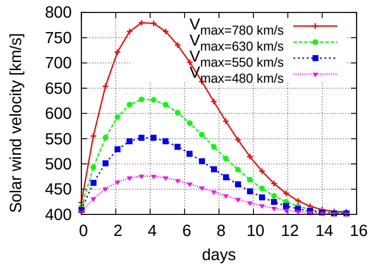

Fig. 4: Changes of the solar wind velocities during the Fd included in the models of the Fd.

IMF, corresponding to assumed solar wind velocities (6), obtained as solutions of the system of equation (4) are presented in Figs. 5-6, respectively. These IMF

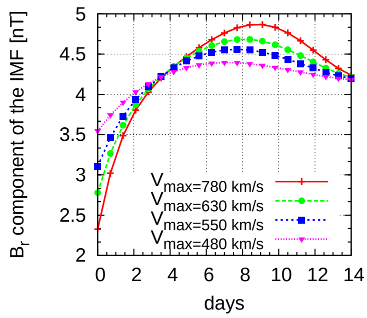

Fig. 5: Azimuthal changes of the B_r component of the IMF at the Earth orbit during the Fd for different solar wind velocities

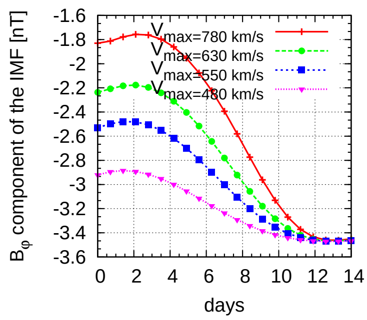

Fig. 6: Azimuthal changes of the B_φ component of the IMF at the Earth orbit during the Fd for different solar wind velocities

components were included to the equation (2) which was transformed by the implicit finite difference method to the system of the algebraic equation; then this system was solved by the Gauus-Seidel iteration method with the initial and boundary conditions as:

$$\begin{aligned} f|_{R_0=150GV} &= 1 \\ f|_{r=100AU} &= 1 \\ \frac{\partial f}{\partial r}|_{r=0} &= 0 \\ \frac{\partial f}{\partial \theta}|_{\theta=0^o} = \frac{\partial f}{\partial \theta}|_{\theta=180^o} &= 0 \\ f|_{\varphi=\varphi_1} &= f|_{\varphi=\varphi_{L+1}} \\ f|_{\varphi=\varphi_{-1}} &= f|_{\varphi=\varphi_{L-1}} \end{aligned} \quad (7)$$

Changes of the relative density obtained as a solution of the Eq. 2 for various profiles of the solar wind velocity (Fig. 4) and calculated expected power law rigidity spectrum exponents γ of the Fd are presented in Fig. 7 and Fig. 8, respectively. We see that the various levels of the solar the wind velocity- an increase of it, causes a increase of the amplitudes of the simulated Fds (Fig. 7), but stays without any influence on the changes of the rigidity spectrum exponent γ, as it is seen from Fig. 8. This effect confirms our assumption that a change of the rigidity spectrum of Fd is observed only versus the changes of the exponent ν. Moreover, the temporal changes of the expected rigidity spectrum exponent γ in different phases of the Fd does not depend on the level of convection of the GCR stream.

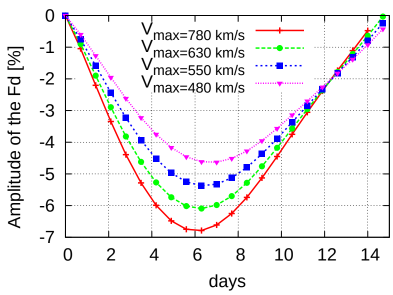

Fig. 7: Changes of the expected amplitudes of the Fd of the GCR intensity for the rigidity of 10 GV based on the solutions of the model of the Fd for different solar wind velocities.

IV. Conclusions

- The relationship between the exponent γ of the rigidity spectrum of the Fd of the GCR intensity and the exponent ν of the PSD of the IMF turbulence (frequency range $f \sim 10^{-6}Hz \div 10^{-5}$ Hz) is observed during the Fd occurred in 9-25 September 2005
- The proposed models reasonably describe the behavior of the exponent γ during the Fd. Theoretical calculations are compatible with the results obtained based on the neutron monitors and ground muon telescopes experimental data and confirms theoretically a dependence of the expected rigidity spectrum exponent γ of the Fd on the exponent ν of the PSD of the IMF turbulence; at the same time γ does not respond to the changes of the solar wind velocity (changes of convection), besides amplitudes of the Fd of the GCR intensity depend on different levels of convection in the rigidity range of $10 \div 50$ GV of GCR.

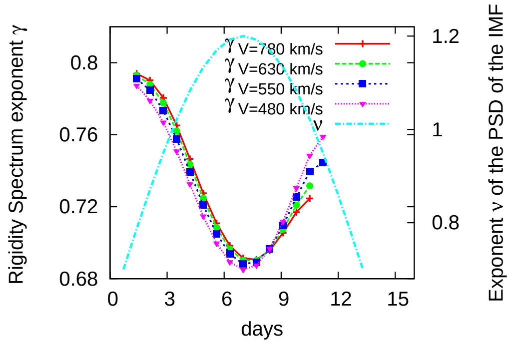

Fig. 8: Temporal changes of the expected rigidity spectrum exponent γ and exponent ν of the PSD of the IMF turbulence for the models of the Fd with different solar wind velocities.

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