Muon Production of Hadronic Particle Showers in Ice and Water

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***Abstract*. One of the neutrino signatures in Cerenkov neutrino detectors are isolated, particle showers induced by neutrinos of all flavors. Hadronic showers can produce muons during the shower development and the appearance of the showers can change significantly by such high-energy muons. We use a modified version of the air shower simulation program CORSIKA for the simulation of the generation of muons in salt water. We discuss how the results can be applied for ice. In addition, a simple analytical model is derived, that provides scaling relations for the muon energy spectrum and its dependence on the primary particle.**

***Keywords*: shower-simulation, muons, neutrino-detection**

I. Introduction

Large neutrino telescopes in ice or water, like IceCube[1], Baikal[2] and ANTARES[3], are in operation or construction. They detect Cerenkov light from particles created by neutrino interactions. The most prominent signature are up-going muons generated by charge-current interactions of muon neutrinos. Another possibility is the search for neutrino induced cascades [4]. Such a search is sensitive to electron and tau neutrinos. In addition, unlike a search for neutrino induced muons, one is not restricted to up-going tracks, since the signature of isolated cascades allows in principle good separation from the down-going atmospheric muon background. A requirement for a search of neutrino induced events is their accurate simulation.

In hadronic cascades also muons can be produced. Due to their track-like light signature they may influence the shape and thus should be simulated and parameterized.

We develop a simple analytical calculation for the produced muon flux (section II). We show the results of a full shower simulation with a modified version of the well-known air-shower simulation program CORSIKA [5] (section III) and use this to parameterize the muon flux (section IV). We conclude in section V.

II. Analytical Model for the Muon Flux

The basic properties of electro-magnetic showers can be understood through the Heitler model [6]. After one interaction length a photon creates an electron-positron pair, and an electron/positron radiates a bremsstrahlung photon. This repeats every interaction length while the energy is distributed over the generated particles, so that more and more particles with lower and lower energies are created. Hadronic showers are more complicated, since in each interaction a wide range of hadronic particles can be created, which through decay increase the complexity further. However, expanding the simple Heitler model helps to get a basic understanding of the muon generation in hadronic cascades.

We follow the extended Heitler model of [7] and consider a hadronic shower generated by a particle of primary energy E_0. In each interaction $N_{\mathrm{mul}} \approx 10$ hadrons are produced. About one third of them will be neutral hadrons like π^0, which will lead to an electromagnetic sub-shower.

So in the generation n we have $N_{\mathrm{H}}(n)$ hadrons with average energy E_{H}:

$$N_{\mathrm{H}}(n) = \left(\frac{2}{3} N_{\mathrm{mul}}\right)^n \quad (1)$$

$$E_{\mathrm{H}}(n) = \frac{E_0}{N_{\mathrm{mul}}^n} \quad (2)$$

Combining this leads to the energy depending number of hadrons per generation

$$\frac{\Delta N_{\mathrm{H}}}{\Delta \log_{N_{\mathrm{mul}}}(E_0/E_{\mathrm{H}})}(E_{\mathrm{H}}) = \left(\frac{E_{\mathrm{H}}}{E_0}\right)^{-\kappa} \quad \text{with } \kappa := 1 + \log_{N_{\mathrm{mul}}} \frac{2}{3} \quad (3)$$

and to the hadronic flux

$$\frac{\mathrm{d}N_{\mathrm{H}}}{\mathrm{d}E_{\mathrm{H}}}(E_{\mathrm{H}}) = -\frac{\ln N_{\mathrm{mul}}}{E_0}\left(\frac{E_{\mathrm{H}}}{E_0}\right)^{-(\kappa+1)}. \quad (4)$$

Next we consider different types of hadrons h. In the following, we will keep using H if a variable is meant for all hadrons. We neglect the different reaction channels for different hadrons and assume a constant branching ratio B_{h} for production of a hadron h, independent of the incident type. (We mainly focus on pions and kaons.)

The muon flux can now be derived as the hadron flux multiplied with the decay probability $P_{\mathrm{h}\to\mu}$ and folded by the energy distribution of the generated muon $\frac{\mathrm{d}n_{\mathrm{h}}}{\mathrm{d}E_\mu}(E_\mu, E_{\mathrm{h}})$:

$$\frac{\mathrm{d}N_\mu}{\mathrm{d}E_\mu}(E_\mu) = \sum_{\mathrm{h}} B_{\mathrm{h}} \int_0^\infty \frac{\mathrm{d}n_{\mathrm{h}}}{\mathrm{d}E_\mu}(E_\mu, E_{\mathrm{h}})\, P_{\mathrm{h}\to\mu} \frac{\mathrm{d}N_{\mathrm{H}}}{\mathrm{d}E_{\mathrm{H}}}(E_{\mathrm{h}})\,\mathrm{d}E_{\mathrm{h}} \quad (5)$$

TABLE I
USED NUMERICAL VALUES FOR PIONS AND KAONS. THEIR CONTRIBUTION A TO THE AMPLITUDE AS WELL AS THE FULL AMPLITUDE ARE PROVIDED.

h	B_{h}	$b_{\mathrm{h}\to\mu}$	α	r_{h}	$A_{\mathrm{h}}\,[\mathrm{GeV}^{-1}]$
π	0.9	1.00	67.1	$5.73 \cdot 10^{-1}$	$20.03 \cdot 10^{-3}$
K	0.1	0.64	9.03	$4.58 \cdot 10^{-2}$	$6.00 \cdot 10^{-3}$
		$A = \sum_{\mathrm{h}} A_{\mathrm{h}}$:			$26.30 \cdot 10^{-3}$

The decay probability of a hadron with mass m_{h} and lifetime τ_{h} and a branching ratio $b_{\mathrm{h}\to\mu}$ for the decay into muons is given by

$$P_{\mathrm{h}\to\mu} = b_{\mathrm{h}\to\mu}\frac{\Lambda}{\lambda_{\mathrm{D}}} = \frac{b_{\mathrm{h}\to\mu}}{1+\alpha_{\mathrm{h}}E_{\mathrm{h}}} \approx \frac{b_{\mathrm{h}\to\mu}}{\alpha_{\mathrm{h}}E_{\mathrm{h}}} \quad \text{with } \alpha_{\mathrm{h}} := \frac{\tau_{\mathrm{h}}}{m_{\mathrm{h}}\lambda_{\mathrm{I}}}, \tag{6}$$

where $\Lambda := \frac{1}{\lambda_{\mathrm{I}}} + \frac{1}{\lambda_{\mathrm{D}}}$, λ_{I} is the interaction length of the hadron and $\lambda_{\mathrm{D}} = \frac{E_{\mathrm{h}}\tau_{\mathrm{h}}}{m_{\mathrm{h}}}$ is the decay length, assuming $E_{\mathrm{h}} \gg m_{\mathrm{h}}$. The approximation holds for $\alpha_{\mathrm{h}}E_{\mathrm{h}} \gg 1$. The probability to create a muon of energy E_μ from the decay of a hadron with energy E_{h} is given by the energy distribution $\frac{\mathrm{d}n_{\mathrm{h}}}{\mathrm{d}E_\mu}(E_\mu, E_{\mathrm{h}})$. Unpolarized meson performing a two-body decay will generate mono-energetic muons isotropic distributed over the directions in their rest-frame (see [8]). This transforms in the laboratory system to a constant distribution between the minimal energy $r_{\mathrm{h}}E_{\mathrm{h}}$ with $r_{\mathrm{h}} = \frac{m_\mu^2}{m_{\mathrm{h}}^2}$ and the maximal energy E_{h}:

$$\frac{\mathrm{d}n_{\mathrm{h}}}{\mathrm{d}E_\mu}(E_\mu, E_{\mathrm{h}}) = \begin{cases} \frac{1}{(1-r_{\mathrm{h}})E_{\mathrm{h}}} & r_{\mathrm{h}}E_{\mathrm{h}} \le E_\mu \le E_{\mathrm{h}} \\ 0 & \text{else} \end{cases} \tag{7}$$

Applying eq. (4), (6) and (7) on eq. (5) we obtain the muon flux

$$\frac{\mathrm{d}N_\mu}{\mathrm{d}E_\mu}(E_\mu) = A\left(\frac{E_0}{\mathrm{GeV}}\right)^{\kappa}\left(\frac{E_\mu}{\mathrm{GeV}}\right)^{-(\kappa+2)} \quad \text{with } A = \frac{\ln N_{\mathrm{mul}}}{\kappa+2}\left(\sum_{\mathrm{h}} B_{\mathrm{h}}\frac{b_{\mathrm{h}\to\mu}}{\alpha_{\mathrm{h}}}\frac{1-r_{\mathrm{h}}^{\kappa+2}}{1-r_{\mathrm{h}}}\right)\frac{1}{\mathrm{GeV}^2}. \tag{8}$$

This results in the numerical values for the amplitude $A = 26.3 \cdot 10^{-3}\,\mathrm{GeV}^{-1}$ and for the exponent according to eq. (3) $\kappa = 0.824$. The values used for pions and kaons are summarized in Table I.

III. CORSIKA SIMULATION

For the simulation we used a modified version of CORSIKA based on the official version 6.2040 which enables shower simulation in salt water (see [9]). The used interaction models are Gheisha for low energies and QGSJet 01 for high energies. We simulated 1000 showers at primary energy $E_0 = 1\,\mathrm{TeV}$, 1000 at $10\,\mathrm{TeV}$, 100 at $100\,\mathrm{TeV}$ and ten at $1\,\mathrm{PeV}$ with a proton as primary particle.

The used configuration (Fig. 1) is a CORSIKA observation level $9\,\mathrm{m}$ behind the interaction point. Here the shower is expected to be fully developed, while only the very low energy muons will already be decayed.

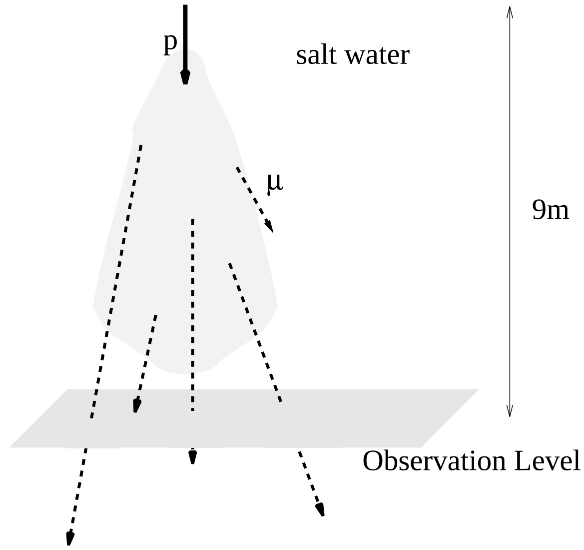

Fig. 1. CORSIKA configuration: the incoming proton p interacts $9\,\mathrm{m}$ above the observation level. The produced muon μ will roughly have the same direction as the incoming particle and are recorded, as they pass through the observation level.

IV. DERIVED MUON FLUX

With the simulation data we calculate the muon flux. To make this comparable to our model we need to calculate the muon energy at generation E_μ from the muon energy at observation level E_μ^{obs} given in the simulation. This is a small correction, important only for low energetic muons. Muons that are most interesting for us are those with track-lengths above about $10\,\mathrm{m}$, namely those with a range bigger than the typical shower size. These muons have energies above $3\,\mathrm{GeV}$, which is high enough, to reduce systematic error due to the energy correction.

The energy loss can be approximated by:

$$\frac{1}{\varrho}\frac{\mathrm{d}E_\mu}{\mathrm{d}x} = -a - bE_\mu \tag{9}$$

where the medium density is $\varrho = 1.02\,\mathrm{g\,cm^{-3}}$ and the interaction constants are $a = 2.68\,\mathrm{MeV cm^2\,g^{-1}}$ and $b = 4.7 \cdot 10^{-6}\,\mathrm{cm^2\,g^{-1}}$ (see [10]). Solving this provides the formula for the energy at generation:

$$E_\mu = \left(E_\mu^{\mathrm{obs}} + \frac{a}{b}\right)\mathrm{e}^{\varrho b x} - \frac{a}{b} \tag{10}$$

Here we choose $x = 7\,\mathrm{m}$, the distance from first interaction point to the observation level reduced by about three radiation length [10].

We show the normalized flux $\frac{E_\mu^2}{E_0^\kappa}\frac{\mathrm{d}N_\mu}{\mathrm{d}E_\mu}(E_\mu)$ which should follow according to eq. (8) a power law with small primary energie dependency (Fig. 2). The power law was fitted for the curves individually (Tab. II). This leads to the averaged parameterization:

$$\frac{\mathrm{d}N_\mu}{\mathrm{d}E_\mu} = A\left(\frac{E_0}{\mathrm{GeV}}\right)^{\kappa}\left(\frac{E_\mu}{\mathrm{GeV}}\right)^{-(2+\kappa)}$$
$$A = (3.5 \pm 0.5\,(\mathrm{stat}) \pm {}^{6.5}_{2.0}\,(\mathrm{sys}))\cdot 10^{-3}\,\frac{1}{\mathrm{GeV}}$$
$$\kappa = 0.97 \pm 0.07\,(\mathrm{stat}) \pm 0.12\,(\mathrm{sys}). \tag{11}$$

The systematics were estimated by using different x-values for the energy correction.

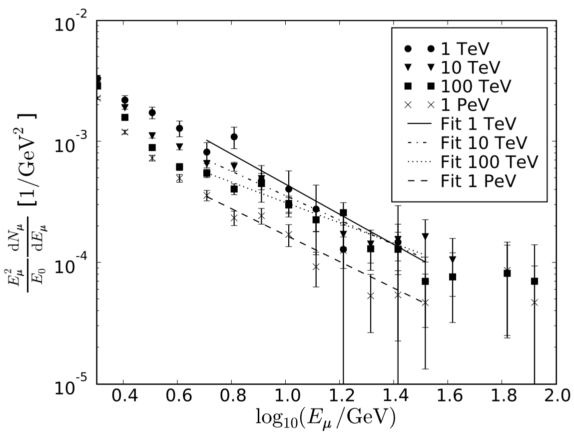

Fig. 2. The muon flux $\frac{\mathrm{d}N_\mu}{\mathrm{d}E_\mu}$ is shown as function of muon energy E_μ. We multiplied by $\frac{E_\mu^2}{E_0}$ to remove the primary energy dependency by some extend and improve readability. The results of the fit are given in Table II.

Using the integral representation (Fig. 3), we can see that on average a hadronic cascade of $100\,\mathrm{TeV}$ produces e.g. about one muon with an energy above $10\,\mathrm{GeV}$. Such a muon would have a track length of about $36\,\mathrm{m}$.

CORSIKA provides the information if a muon was generated from a pion. Using this information, Fig. 4 shows the number of pion produced muons over all muons as a function of the energy. As expected, other

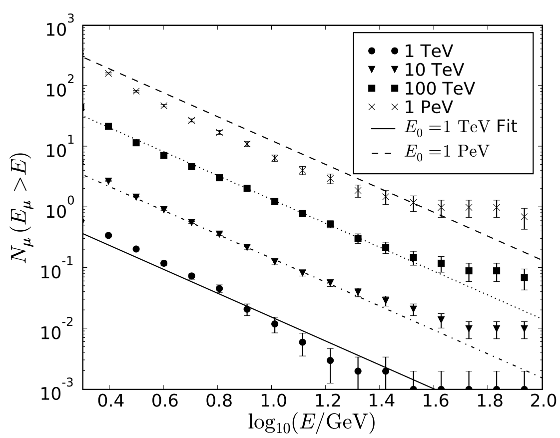

Fig. 3. Integral muon flux $N_\mu(E_\mu > E)$. The fit is taken from the differential muon flux (Fig. 2 and Table II).

TABLE II
PARAMETERIZATION OF MUON FLUX: THE PARAMETERIZATION GIVEN IN (11) WAS FITTED TO THE MUON FLUX INDIVIDUALLY FOR THE DIFFERENT PRIMARY ENERGIES (S. FIG. 2).

E_0	$10^3 \cdot A\,[\frac{1}{\mathrm{GeV}}]$		κ	
$10\mathrm{TeV}$	4.731	±0.995	1.007	±0.104
$100\mathrm{TeV}$	3.020	±0.703	0.840	±0.113
$1\mathrm{TeV}$	9.457	±6.792	1.242	±0.371
$1\mathrm{PeV}$	3.089	±0.965	1.091	±0.156
average	3.490	±0.492	0.971	±0.068

production mechanisms (e.g. kaons) become more important with higher energy. However the constant hadron fractions B_h are a reasonable approximation. Our simple model with only pion and kaon ($B_K = 0.1$) predicts a constant $\frac{N_\mu^\pi}{N_\mu} = 77\,\%$.

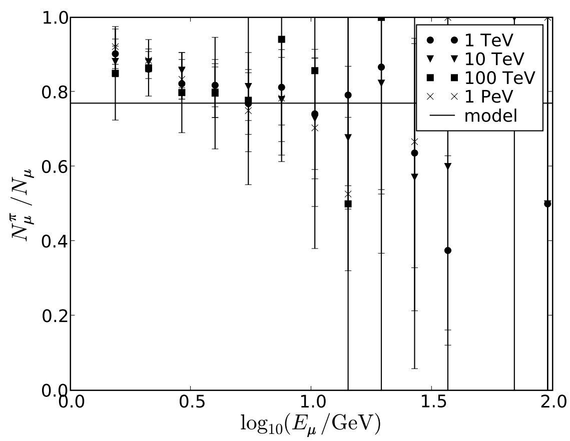

Fig. 4. Muon parent: the ratio of number of muons produced by pions N_μ^π over all number of muons N_μ for each muon energy E_μ is shown. A slight increase with energy of muons produced by other parent hadrons can be observed. However, the comparison to the model shows that the constant fraction is a reasonable approximation.

V. DISCUSSION AND CONCLUSION

We developed an analytical model describing the muon production in hadronic cascades, showing that it follows a power law with exponent $-(2+\kappa)$. The amplitude scales linear with the medium density ϱ and the primary energy E_o^κ. This is an effect of approximately $10\,\%$ in amplitude for changing from salt water to ice.

We compared this model to a full shower simulation with a modified CORSIKA version. The used setup forced us to correct for energy losses. To minimize the influence of the correction and because of the short track length of low energetic muons we focus on energies above $3\,\mathrm{GeV}$.

We fitted a power law to the results (Fig 2 and Table II) and get the parameters with systematic errors due to the energy correction: $A = (3.5 \pm 0.5\,(\mathrm{stat}) - 2.0 + 6.5\,(\mathrm{sys}))\cdot 10^{-3}\,\mathrm{GeV}^{-1}$ and $\kappa = 0.97 \pm 0.07\,(\mathrm{stat}) \pm 0.12\,(\mathrm{sys})$. Around an energy of $E_\mu = 10\,\mathrm{GeV}$ and a primary energy $E_0 = 10\,\mathrm{TeV}$, the value most important to us, the analytical model predicts a flux a factor three higher than the flux from the simulation results.

This study could be checked with a newer modified CORSIKA version [11], which would provide direct results for ice and the possibility to compare different hadronic interaction models and increase the statistic.

The parameterization of muon production in hadronic cascades that we provide can be used to simulate more accurately neutrino induced cascades.

VI. ACKNOWLEDGEMENTS

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