Multiple scattering of the fluorescence light emitted by extensive air showers in the atmosphere

Maria Giller and Andrzej Śmiałkowski

Department of High Energy Astrophysics, University of Łódź, Poland

***Abstract*. Fluorescence light emitted by a shower may be scattered away and back to the field of view of a detector distorting the shower image. Here we derive analytical formulae for the number of such photons. The corresponding integrals have to be calculated numerically, so, for practical reasons, we express our results as functions of geometrical and atmospheric parameters. Our results show that the effect is a bit larger than previously thought. The scattered light superimposes on the shower image in the direct light and has to be subtracted in order to calculate correctly the primary energy of a shower.**

***Keywords*: extensive air showers, fluorescence light, cosmic rays**

I. Introduction

Charged particles of an extensive air shower emit fluorescence and Cherenkov light. The fluorescence light is emitted isotropically from any point of a shower track, whereas Cherenkov light propagates more or less in the same direction as the emitting particles. One of the methods for determining the primary energy of a shower is detecting the emitted light as a function of time and height of the emission, while looking at a shower from a side (as in the experiments Fly's Eye, HiRes and the Pierre Auger Observatory). A typical, time integrated image of a high energy shower is a sequence of hit photomultipliers on a telescope camera. The width of this image is determined by the lateral distribution of electrons in the shower and by the lateral distribution of Cherenkov light scattered at the observed shower element towards the detector. Moreover, the emitted photons are scattered on their way from the shower to the detector and smear the image so that the reconstruction of the number of the fluorescence photons (proportional to the energy deposit along the shower) becomes more complicated. In this paper we study this effect. Namely, we calculate the angular distribution of light in an instantaneous image of a shower assuming that it can be approximated by a point source of isotropic light moving with light velocity. In this way we are modelling the fluorescence light emitted by a distant shower. The emitted light is scattered in the atmosphere and arrives at the detector at the same time as the direct light (not scattered).

The problem of the instantaneous images of showers has been treated by Roberts [3] and Pękala et al [4] by a Monte Carlo method. Here we show how to solve it analytically (although integrals have to be calculated numerically). We have already tackled this problem analytically in our previous papers [1] and [2], but from a slightly different point of view. We concentrated there more on time integrated shower images and on calculating the amount of the multiply scattered light in a fixed field of view. Moreover, some of our calculations were based on an approximate assumption (see the second generation treatment in [2])

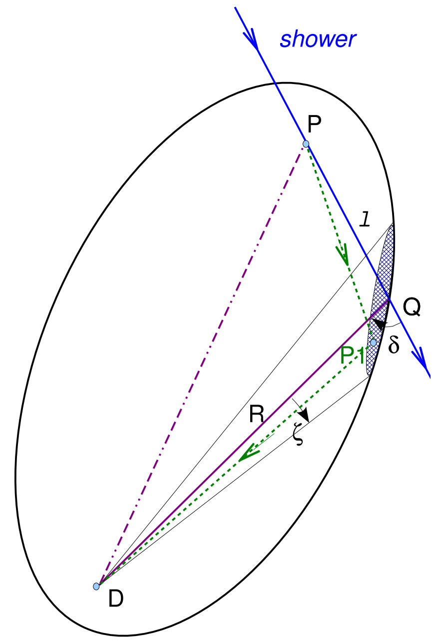

Fig. 1: Geometry for the first generation. Direct (unscattered) light produced at Q arrives at the detector at D. Photons produced *above* point Q (e.g. at P) and scattered on the shaded surface of the ellipsoid (with foci at D and P) at point P_1 on figure arrive at the same time.

II. The Method

We consider a point source of isotropic light moving through the atmosphere with light velocity c, corresponding to a distant shower emitting fluorescence light. Our aim is to calculate analytically its instantaneous image i.e. the angular distribution of light arriving at a detector within a small time interval Δt, assuming that the photons are scattered in the atmosphere by the Rayleigh (molecular) and Mie (aerosol) processes. For the sake of our calculations we adopt an atmosphere corresponding roughly to the Pierre Auger Observatory site in Malargue (Argentina): the mean free path length increases exponentially with height above the observation level (∼1450 m a.s.l.) with different scale heights for Rayleigh and Mie scatterings:

$$\lambda(h) = \lambda(0) \cdot e^{h/H} \tag{1}$$

where $\lambda(0) = 18\,km$, $H = 9\,km$ for Rayleigh and $\lambda_M(0) = 15\,km$, $H = 1.2\,km$ for Mie scatterings.
It can be easily shown that for an exponential atmosphere with a scale height H the effective mean free path between two points A and B, located at height h_A and h_B correspondingly, equals

$$\lambda_{AB} = \lambda(h_A)\frac{Z_B - Z_A}{1 - e^{-Z_B - Z_A}} \tag{2}$$

where $Z_{A,B} = h_{A,B}/H$.
The geometry of the situation is presented in Fig.1. The light source (a shower) moves along the line PQ. The telescope field of view is centred at point Q. At time $t = QD/c = R/c$ after the source passed this point, the direct (unscattered) light emitted at Q within a small time interval Δt arrives at the detector located at point D. At the same time some scattered light emitted *above* Q arrives at the detector as well.
The scattered light consists of photons scattered exactly once, twice and so on. We shall call them the first generation, the second one and so on correspondingly. We will calculate the number of these photons as a function of the geometry and the opening angle of the detector field of view ζ.
Let us consider the photons scattered only once, produced at point P along a path length dl, arriving at the detector at the same time interval Δt as the direct light produced at Q. To arrive within a time bin Δt the direct light must be emitted along a path length equal to $c\Delta t/(1 - cos\delta)$, where δ is the angle between the shower direction and line QD. Thus, the number of direct photons, N_0, arriving at the detector within time bin Δt equals

$$N_0 = C\, c\Delta t/(1 - cos\delta) \cdot \frac{A}{4\pi R^2} e^{-\frac{R}{\lambda_{QD}}} \tag{3}$$

where C equals to the number of the photons produced per unit length, A is effective diaphragm area determining the collection solid angle and λ_{QD} is total effective mean scattering path length on the way from point Q to D (formula (2)).
The scattering points must lie on the surface of the rotational ellipsoid, with the focal points at D and P and the rotational symmetry axis DP. The viewing cone with the opening angle ζ cuts out a region on the surface of the ellipsoid (the shaded area in Fig.1) at which the scattering must take place in order to be detected. Thus, to find the total number of photons undergoing one scattering only, N_1, one has to integrate the number of the scattered photons over this cut out surface for each

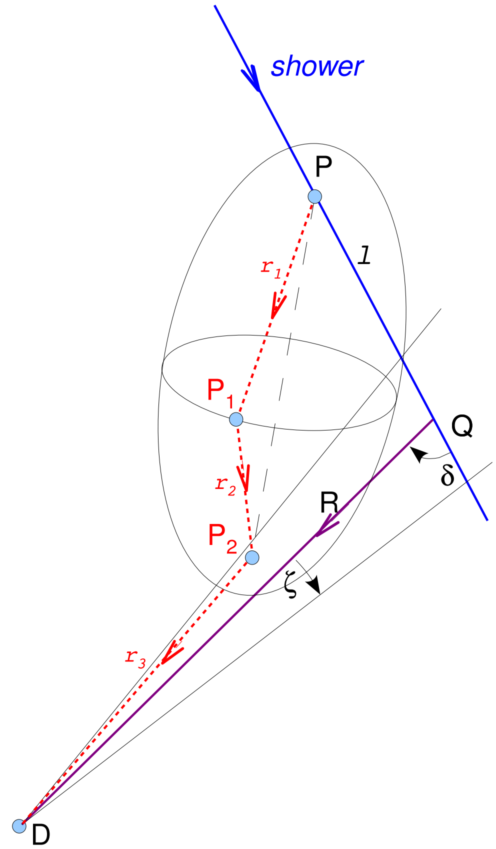

Fig. 2: Geometry for the second generation. For any fixed point P_2 of the second scattering there corresponds an ellipsoid (with foci at P_2 and P) on which surface the first scattering must take place.

path element dl of the source and next integrate over l, the distance from point P to Q.
So, we have

$$N_1 = \int_0^{l_{max}} \int_{\Omega_1} C dl\, f_P(\theta) d\Omega_1 \frac{\Delta r_1}{\lambda_1} f_s(\alpha)\Omega_D \cdot \\ \cdot e^{-(\frac{r_1}{\lambda_{01}} + \frac{r_2}{\lambda_{12}})} \tag{4}$$

where $f_P(\theta)$ is the angular distribution of the emitted light, $d\Omega_1$ is the solid angle determined by point P and the integration area dS on the cut-out surface of the ellipsoid; λ_{01} and λ_{12} are the effective attenuation lengths for the light path $PP_1 = r_1$ and $P_1D = r_2$ respectively; Δr_1 is a path element (along $PP_1 = r_1$) of such length that the light scattered there by the angle α will arrive within time bin Δt: $\Delta r_1/c = \Delta t/(1 - cos\alpha)$, λ_1 is the mean scattering path length at P_1; $f_s(\alpha)$ is the angular distribution of the scattered light; $\Omega_D = A/r_2^2$, where A is the detector area perpendicular to the arriving light (to P_1D).

Now, we shall consider the photons scattered twice. The point of the second scattering must lie somewhere inside the viewing cone. Fixing this point (P_2 in Fig.2) at a distance r_3 from the detector corresponds to fixing

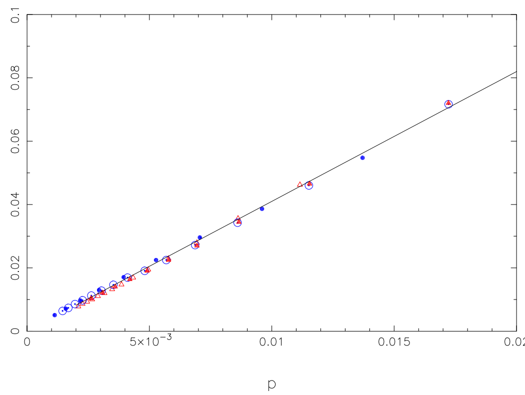

Fig. 3: Ratio of the scattered to direct light, N/N_0 as a function of parameter p (formula (6)) within $\zeta = 1°$ (open symbols) and $\zeta = 2°$ (filled symbols). Distance to shower core $16\,km$ (triangles, red) and $32\,km$ (circles, blue). Showers with angle $\delta = 90°$ (vertical). Line - formula (7).

the value of the remaining photon path length from the emission point P to the point of the first scattering P_1 and from this point to the point of second scattering, P_2. We have that

$$PP_1 + P_1P_2 = R + l - r_3 \tag{5}$$

where $l = PQ$ and $R = DQ$.
In order to assure that the photons scattered twice arrive at D at the same time as the direct light emitted at Q, point P_1 must lie anywhere on the surface of the ellipsoid with foci at P and P_2. Thus, to calculate the number of photons produced at P and scattered twice one has to integrate over the position of the first scattering (the whole surface of the ellipsoid) and then integrate the result over the volume of the viewing cone. Finally, the integration has to be performed over the distance $l = PQ$.
We have shown in our earlier work and will do it here that the contribution of the second generation is on the level of $\sim$ 1% of the total light flux, so we will not consider the higher generations .

III. Results

As the integrations have to be done numerically it is practical to find a convenient parametrization of the results. We shall parametrize the ratio N/N_0, where N is the total scattered light (in our case $N = N_1 + N_2$). Our calculations show that the amount of the scattered light should depend on the product $R\zeta$. It should also depend on the mean free path for scattering at point Q (the main contribution comes from a part of the shower not far from above this point). As it turns out, the ratio N/N_0 depends also on λ_{QD}, the effective mean free path along distance $R = QD$.
Fig.3 shows the ratio of the number of scattered photons, N, to that of the direct light, N_0, arriving simultaneously at the detector, as a function of a parameter p, where

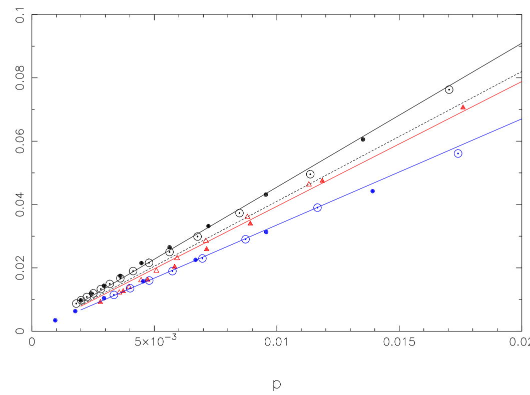

Fig. 4: The same as for Fig.3, but showers with $\delta = 60°$ (red, blue) and $\delta = 120°$ (black, upper points). Dashed black line represents fit to vertical shower from previous figure. Other lines are fits to points according to (7).

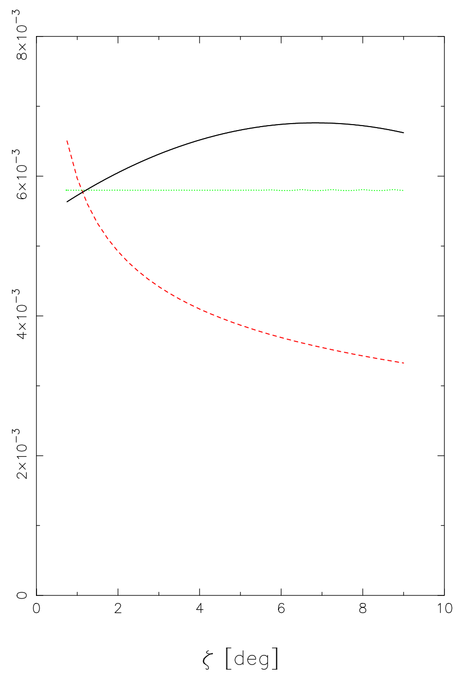

Fig. 5: Ratio of the scattered to direct light, $\Delta N/N_0$, collected within rings with radius ζ and width $\Delta\zeta = 0.25°$. Curves corresponds to results obtained with parametrisations from Monte-Carlo works by Roberts (dashed red line) and Pękala (dotted flat green line). Our results (first and second generation) are represented by the black solid line.

$$p = R\zeta/(\lambda_Q\lambda_{QD})^{0.77} \tag{6}$$

Points correspond to different elevation angles of the field of view, different distances of the shower core from the detector and $\delta = \pi/2$ (vertical showers). It can be seen that there is a very good proportionality $N/N_0 \sim p$. However, for other angles δ the ratio N/N_0 changes. In particular, when the inclination angle δ becomes smaller

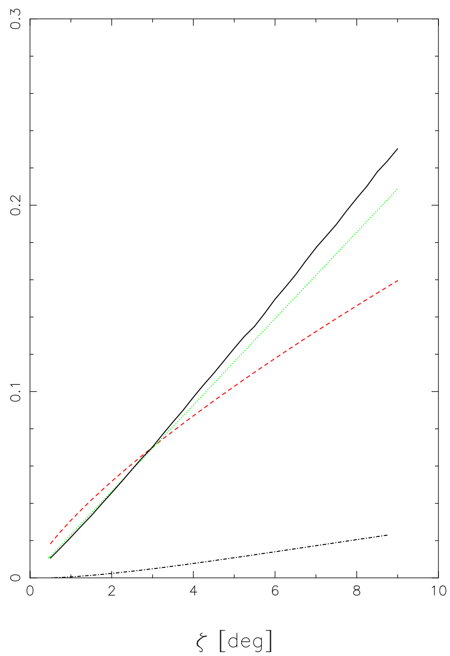

Fig. 6: Ratio of the scattered to direct light $N(<\zeta)/N_0$ integrated within angle ζ. Our results (first and second generation) are represented by the black solid line. Bottom dashed line shows contribution of the second generation. Other lines are parametrisations from Monte-Carlo works: Roberts - dashed red line and Pękala - dotted green line.

than $\pi/2$, the number of direct photons begins to grow quickly (as $1/(1-cos\delta$) and the ratio diminishes, as it is seen in Fig.4. So, our parametrization taking into account the dependence on δ is:

$$N/N_0 = 4.1 \cdot p \cdot (1 - cos\delta)^{0.17(R/12-1)} \quad (7)$$

where R is in km. It can bee seen that it is quite good for the regions where N/N_0 is large. It underestimates the effect only there where it is below 2%.

The dependence on the opening angle ζ of the detector field of view, for a vertical shower 24 km away from the detector is shown in Fig.5. The elevation angle of the centre of the field of view is $10°$. The vertical axis represents the ratio of the number of the scattered photons, ΔN (first and second generation), arriving at an angle ζ within $\Delta\zeta = 0.25°$, to the direct light N_0. ΔN is practically constant over the region of the considered angles, as it should be if $N(<z) \sim \zeta$.

The agreement with the parametrization of Pękala et al is very good, although Roberts underestimated the effect at higher angles. Fig.6 shows the scattered light integrated within angle ζ. It shows that the $N(<\zeta)/N_0$ grows with ζ without saturation. It is not difficult to understand this while looking at Fig.1 and Fig.2 - the collection region of the scattered light grows with ζ increasing N_1, whereas N_0 stays constant.

IV. Conclusions

We have obtained analytically (with numerical calculations of integrals) the angular distribution of the scattered light arriving at the same time as a direct light from an observed shower. The main contribution to the multiply scattered light are photons scattered only once. Thus, the picture of a diffusion of photons in the atmosphere, evoked by Roberts, is not the right one. Our calculations are in a very good agreement with the parametrisation of Pękala et al of their Monte-Carlo results. Roberts, however, underestimated this effect for angles $\zeta > 2°$.

We also indicated that the relative contribution of the scattered light diminishes for decreasing shower inclination δ because of a rather rapid increase of the direct light arriving within a fixed time bin.

V. Acknowledgements

This work was stimulated by the participation of the authors in the Auger Collaboration. So, we thank the Collaboration for fruitful discussions.

References

[1] Giller M., Śmiałkowski A. ,2005, Proceedings of the 29th International Cosmic Ray Conference, Pune, India
[2] Giller M., Śmiałkowski A., 2007, Proceedings of the 30th International Cosmic Ray Conference, Merida, Mexico
[3] Roberts M. D., 2005, J. Phys. G: Nucl. Phys. 31, 1291
[4] Pę kala et al, 2009, Nucl. Instr. and Meth. in Phys. Res. A, in press