Accelerated ions and selfexcited Alfvén waves at the Earth's bow shock

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***Abstract*. The diffuse energetic ion event and related Alfvén waves upstream of the Earth's bow shock, measured by AMPTE/IRM satellite on September 29, 1984 was studied within the quasilinear approach. It is shown that at the observed level of Alfvénic turbulence in the solar wind steady state ion and Alfvén wave spectra are established during the time period of about 1000 s. Alfvén waves excited by accelerated ions are confined within the frequency range $(10^{-2}-1)$ Hz and their spectral peak with the wave amplitude $\delta B \sim B$ comparable to the interplanetary magnetic field value B corresponds to the frequency 2×10^{-2} Hz. High frequency part of wave spectrum undergoes absorption by thermal protons. Calculated accelerated ion spectra and associated Alfvén wave spectra are consistent with the experiment.**

***Keywords*: particle diffusive shock acceleration, Alfvén waves generation, Earth's bow shock**

I. Introduction

According to the numerous measurements performed near the Earth's bow shock an intense generation of the so-called diffuse population of energetic ions accompanied by a significant increase of Alfvénic turbulence always takes place during the periods when the interplanetary magnetic field (IMF) B is nearly radial (see, e.g., [1]). For such an IMF, the Earth's bow shock is quasi-parallel at its nose, that provides the most favorable conditions for the diffusive shock acceleration of charged particles.

The consistent description must include the generation of Alfvén waves by accelerated ions, which in turn, determines a self-consistent particle diffusion coefficient.

Here we apply the quasilinear theory of ion acceleration at the Earth's bow shock [2] to the diffuse energetic ion event and related Alfvén waves, measured by AMPTE/IRM satellite on September 29, 1984.

II. Model

Since the diffuse component is observed when the shock is quasi-parallel, we assume the IMF $\boldsymbol{B}$ to be directed radially, just as the solar wind velocity $\boldsymbol{u}$, along the x axis of the reference frame associated with the bow shock (to be more precise, with the nose of the shock, which in our model is assumed to be plane perpendicular to the solar wind velocity). Since the ions are highly magnetized ($\kappa_\parallel \gg \kappa_\perp$), the equation for their distribution function $f(x,v,t)$ upstream of the bow shock ($x<0$) can be written in the form

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_\parallel \frac{\partial f}{\partial x}\right) - u\frac{\partial f}{\partial x} - \frac{f}{\tau_\perp}, \qquad (1)$$

where $\kappa_\parallel$ and $\kappa_\perp$ are the parallel and perpendicular relative to the IMF ion diffusion coefficients and v is the ion speed. We neglect the Alfvén speed $c_a = B/\sqrt{4\pi\rho}$ compared with the solar wind speed, because $u \gg c_a$. Here $\rho = (N + 4N_\alpha)m_p$ is the solar wind density, N and N_α are the proton α-particle number density in the solar wind respectively, m_p is the proton mass. The last term in this equation describes the escape of particles from the acceleration region due to their diffusion across the IMF lines on a characteristic time scale $\tau_\perp = a^2/\kappa_\perp$. Here a is the effective transverse size of the shock, which effectively accelerates ions, so that the acceleration region is determined by the relation $y^2+z^2 \le a^2$. The size of the acceleration region is taken to be $a = 3.2\ R_E$, where R_E is the Earth's radius.

The distribution of accelerated ions in the downstream region is assumed to be close to the uniform. In this case the solution of the problem does not depend upon the downstream parameters.

At the shock front, situated at $x = 0$, Eq. (1) is supplemented by the boundary condition

$$\frac{u}{q} v \frac{\partial f}{\partial v} + \kappa_\parallel \frac{\partial f}{\partial x} = Q_0, \qquad (2)$$

where

$$Q_0 = u\frac{N_{inj}}{4\pi v_{inj}^2}\delta(v - v_{inj})H(t) \qquad (3)$$

is the source that provides the injection of $N_{inj} = \eta N$ particles with the speed v_{inj} into the acceleration from the each unit volume of the gas intersecting the shock front, η is the injection rate, $q = 3\sigma/(\sigma-1)$. The initial time moment $t = 0$ corresponds to the time when the IMF direction changes sharply so that it becomes quasi-radial, while the bow shock becomes quasi-parallel. Note that Eq.(1-3) correspond to a pure parallel shock. In such a case particle acceleration takes place uniformly within the acceleration region $y^2 + z^2 \le a^2$ going monotonically towards the steady state. In the actual situation IMF line makes some angle ψ with the solar wind speed. As a given IMF line makes initial contact with the bow shock and moves across the nose (acceleration region) toward the opposite side the accelerated ion populating the IMF line evolves toward the steady state. The observer (satellite) situated somewhere upstream of the shock see the unsteady particle population, which corresponds to the duration of the acceleration equals to the connection time of the observers IMF line with the shock nose $t \propto a/(u \sin\psi)$.

We adopt here the energy of the injected protons $\varepsilon_{inj} = mv_{inj}^2/2 = 5$ keV. Besides the protons, we take into account the acceleration of α-particles. The energy of the injected α-particles is taken to be $\varepsilon_{inj} = 20$ keV.

Since the bow shock is the only source where the particles are injected into acceleration, the problem should be solved for the initial and boundary conditions

$$f(x, v, t = 0) = 0, \qquad f(x = -\infty, v, t) = 0, \quad (4)$$

which imply the absence of background particles from the energy range under consideration in the solar wind.

The diffusion of particles is due to their resonant interaction with Alfvén waves, that provide the diffusion coefficients [3]

$$\kappa_{\parallel} = \frac{v^2 B^2}{32\pi^2 \omega_B E_w\left(k = \rho_B^{-1}\right)}, \quad (5)$$
$$\kappa_{\parallel}\kappa_{\perp} = \rho_B^2 v^2/3.$$

where $\rho_B = v/\omega_B$ is the gyroradius, $\omega_B = ZeB/(Amc)$ is the gyrofrequency, m and e are the proton mass and charge, c is the speed of light, Z and A are the ion charge and mass numbers, and $E_w(k) = d(\delta B^2/8\pi)/d\ln k$ is the Alfvén wave energy density per logarithm of the wave number k.

The background spectrum of Alfvén waves in the solar wind,

$$E_{w0}(k) = E_w(x = -\infty, k, t = 0), \quad (6)$$

is a mixture $E_w = E_w^+ + E_w^-$ of waves propagating away from and toward the Sun relative to the solar wind with energy densities E_w^+ and E_w^- respectively.

The wave spectrum E_{w0} is modified near the shock due to the Alfvén waves generation and absorption by accelerated particles and due to their damping on thermal protons according to the wave transport equation

$$\frac{\partial E_w^{\pm}}{\partial t} + u\frac{\partial E_w^{\pm}}{\partial x} = \mp\Gamma E_w^{\pm} - L, \quad (7)$$

where

$$\Gamma(k) = \frac{32\pi^3 c_a}{kc^2 v^2}\sum_s \frac{(Ze)^2}{Am}\kappa_{\parallel}(v = \omega_B/k)\times$$
$$\times\int_{v_{\min}}^{\infty} dv v^3\left(1 - \frac{\omega_B^2}{k^2 v^2}\right)\frac{\partial f}{\partial x} \quad (8)$$

is the growth (damping) rate of the waves propagating along (opposite) the x-axis, due to accelerated particle distribution [3]; $v_{\min} = \max(v_{inj}, \omega_B/k)$; "s" refers the type of ion (for the simplicity we omit this index in the corresponding quantities); L is the wave damping rate. Note that the expression (8) for the wave growth rate has more appropriate form compared with what was initially derived by Lee [4].

We use equal amount of oppositely propagating waves $E_{w0}^+ = E_{w0}^-$, that is consistent with the measurements at the Earth's orbit [5], [6].

According to the experiment [7] the background Alfvén wave frequency spectrum $E_{w0}(\nu) = E_{w0}(k = 2\pi\nu/u)/\nu$ at the Earth's orbit has a power-law form $E_{w0}(\nu) \propto \nu^{-\lambda}$ with a break at $\nu_b \sim 0.1$ Hz so that spectral index $\lambda \approx 3/2$ at $\nu < \nu_b$ and $\lambda \approx 3$ at $\nu > \nu_b$. Here $\nu = ku/2\pi$ is the frequency seen by a stationary observer. The break frequency $\nu_b = k_b u/2\pi$ can be determined from the relation [7]

$$k_b = \omega_B/\left(c_a + v_T\right), \quad (9)$$

where $v_T = \sqrt{k_B T/m}$ is the thermal proton speed; T is the proton temperature, and k_B is the Boltzmann constant.

The break in the Alfvén wave spectrum is due to their damping on thermal protons. It is described by the damping term in the wave transport equation (7), which can be represented in the form

$$L = \Gamma_p(E_w^{\pm} - E_{w0}^{\pm}), \quad (10)$$

where

$$\Gamma_p(k) = m_1\omega_B\left[\left(\frac{kc}{\omega_p}\right)^2\right]^{m_2}\exp\left[-m_3\left(\frac{\omega_p}{kc}\right)^2\right], \quad (11)$$

$\omega_p = \sqrt{4\pi Ne^2/m}$ is the proton plasma frequency, $m_1 = 0.60\beta^{0.36}$, $m_2 = 0.77\beta^{0.03}$, $m_3 = 0.32/\beta^{0.65}$, and $\beta = 8\pi k_B NT/B^2$ is the plasma parameter [8].

The formulated problem (1) – (8) is solved numerically.

III. Results and discussion

We apply our model to describe the diffuse energetic ion event and related Alfvén waves upstream of the Earth's bow shock, measured by AMPTE/IRM satellite on September 29, 1984 at time period 06:42-07:22 UT. We use the values of solar wind parameters, measured during this period of time: solar wind speed $u = 414$ km s^{-1}, IMF strength $B = 5\times10^{-5}$ G, proton number density $N = 2.2$ cm^{-3} and α-particles number density $N_\alpha = 0.37$ cm^{-3}.

During the considered period the proton temperature is $T = 7\times10^4$ K, that gives the break frequency $\nu_b = 0.35$ Hz. Wave damping leads to the steep Alfvén wave spectrum $E_{w0}(\nu) \propto \nu^{-3}$ at $\nu > \nu_b$ (see below). In order to fit the data at $\nu = 10^{-3} - 10^{-2}$ Hz and at $\nu > \nu_b$ the background Alfvén wave spectrum is taken in the form

$$E_{w0}(\nu) = E_0(\nu/\nu_{inj})^{-\lambda} \quad (12)$$

with the amplitude $E_0 = 1.2\times10^{-12}$ erg cm^{-3} and spectral index $\lambda = 1.15$ values.

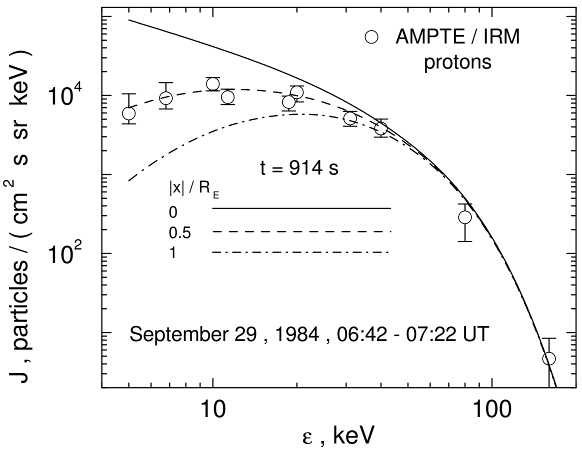

Fig. 1. The intensity of accelerated protons as a function of their energy, calculated for three different distances upstream of the shock at time moment $t = 914$ s from the beginning of the intense acceleration. The experimental data measured by AMPTE/IRM satellite for September 29, 1984, 06:42-07:22 UT event are shown as well.

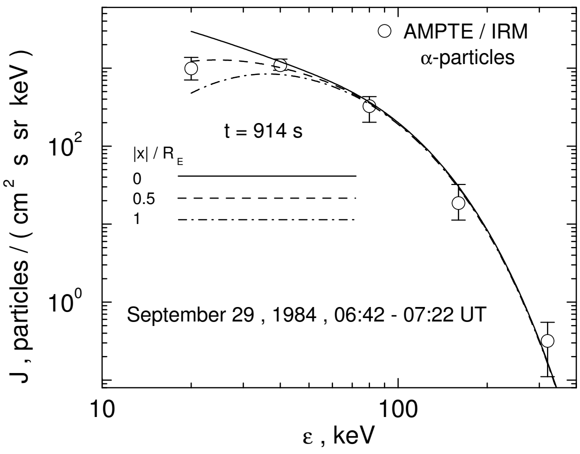

Fig. 2. The same as in Fig.1, but for α-particles.

The value of the shock compression ratio $\sigma = 3.8$ is adopted here.

We adopt injection rate $\eta = 1.3 \times 10^{-2}$ for protons and $\eta = 10^{-2}$ for α-particles. These values provide the best fit to the measured ion intensities.

Using the the model bow shock [9] for the measured solar wind parameters and satellite coordinates we get the connection time $t = 914$ s. The calculations which we present below corresponds to the time moment $t = 914$ s from the beginning of the acceleration process.

We present in Fig. 1 the differential (in respect of kinetic energy $\varepsilon = mv^2/2$) intensity of accelerated protons

$$J(\varepsilon) = v^2 f(v,t)/m, \tag{13}$$

calculated for three different distances upstream of the shock $\Delta x = |x|$ and for $t = 914$ s. Note that accelerated ion population at $t = 914$ s is already very close to their equilibrium. During the observational period the mean distance between the satellite and the bow shock is $\Delta x \approx 1\ R_E$, if the bow shock form corresponds to the Fairfield model [9]. According to Fig.1 calculated proton spectrum is consistent with the experiment if the actual mean distance is $\Delta x \approx 0.5\ R_E$. Calculated proton spectrum corresponding to the distance $\Delta x = 1\ R_E$ is essentially below the experiment at energies $\varepsilon < 20$ keV.

We have similar situation with the energy spectra of accelerated α-particles as one can see in Fig. 2: experiment agrees with calculation corresponding $\Delta x = 0.5\ R_E$, eventhough the discrepancy between the calculated spectrum for $\Delta x = 1\ R_E$ and experiment is not significant in this case.

The calculated Alfvén wave spectra $E_w(\nu) = E_w(k = 2\pi\nu/u)/\nu$ for the same three different distances and for the same time moment are presented in Fig. 3 together with the experimental data. Note that the wave spectrum at frequencies $\nu < 10^{-2}$ Hz as well as at $\nu > 0.35$ Hz for large distances $\Delta x > 1R_E$ is not affected by accelerated ions. It is clearly seen that adopted background spectrum $E_{w0}(\nu)$, shown in Fig.3 by dotted line, fits the data at $\nu = 10^{-3} - 10^{-2}$ Hz and at $\nu > \nu_b$. One can see that calculated wave spectrum corresponding to the distance $\Delta x = 0.5\ R_E$ from the shock front in satisfactory way fits the data. Note that the wave spectrum corresponding to the distance $\Delta x = 1\ R_E$ is also consistent with the experiment.

Note that protons with energies $\varepsilon > 5$ keV are scattered by waves with frequencies $\nu \le \nu_{inj}$, where $\nu_{inj} = \omega_B u/(2\pi v_{inj}) \approx 0.05$ Hz. The low-frequency part of the wave spectrum ($\nu < \nu_{inj}$) reflects the shape of the accelerated particle spectrum. The wave damping on thermal ions, which is described by the source term L in the wave transport equation, leads to the steepening of the wave spectrum above the break frequency $\nu_b \approx 0.35$ Hz.

A distinctive feature of the dynamics of Alfvén waves is a monotonic increase in their energy at all frequencies. This distinguishes it from the wave dynamics, calculated with the wave growth rate derived by Lee [4]. In the latter case, the growth of the wave amplitude is essentially non monotonic, so that the wave amplitude at intermediate times exceeds significantly the subsequently established steady state [10].

The overall energy densities of the Alfvén waves excited by accelerated particles

$$W = \int_0^\infty [E_w(x=0,\nu) - E_{w0}(\nu)]d\nu, \tag{14}$$

at the shock front ($x = 0$) reach the value $W = 0.83 E_B$, where $E_B = B^2/8\pi$ is the IMF energy density.

The pressure of accelerated particles

$$P = \frac{4}{3}\pi \sum_s Am \int_{v_{inj}}^\infty dv v^4 f_s(v), \tag{15}$$

reach the value $P = 0.13\rho u^2$. It is small compared with the ram pressure ρu^2, that justifies the neglect of the shock modification by the particle backreaction.

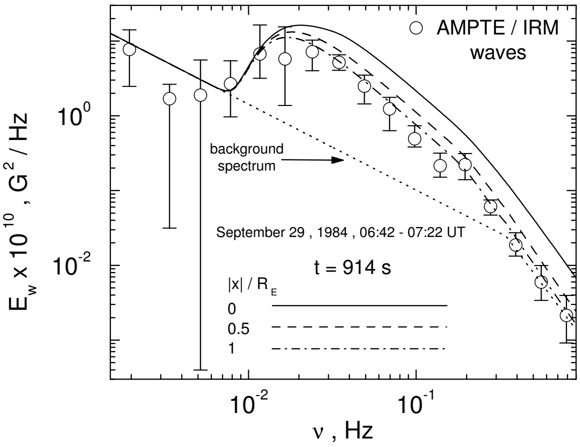

Fig. 3. The Alfvén wave spectra, calculated for three different distances upstream of the shock at time moment $t = 914$ s from the beginning of the intense acceleration. The experimental data measured by AMPTE/IRM satellite for September 29, 1984, 06:42-07:22 UT event are shown as well. Background Alfvén wave spectrum is represented by the dotted line.

IV. Summary

Quasilinear model, developed to describe the diffuse energetic ion events and related Alfvén waves upstream of the Earth's bow shock, was applied to September 29, 1984, 06:42-07:22 UT event detected with instruments on AMPTE/IRM satellite. Calculated spectra of accelerated ions and associated Alfvén waves very well agree with the experiment if the actual distance between the satellite and the shock is $\Delta x = 0.5\ R_E$. Since the distance estimate based on the Fairfield model [9] gives $\Delta x = 1\ R_E$ it is not clear whether this is a real discrepancy between the theory and experiment or it is a result of the uncertainty of the Fairfield model, which we used for determination of the bow shock position.

According to our calculations, the accelerated ions whose spectrum lies in the energy range $5 - 200$ keV intensively excite Alfvén waves with frequencies $\nu > 10^{-2}$ Hz with the spectral maximum at the frequency $\nu \approx 2 \times 10^{-2}$ Hz. Wave amplitudes $\delta B \sim B$ are comparable to the IMF strength B. The damping of Alfvén waves on thermal ions leads to spectrum steepening above the break frequency $\nu_b = 0.35$ Hz.

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