About the cosmic-ray spectrum around the knee

M. Shibata*

Faculty of Engineering, Yokohama National University, Yokohama 240-8501, Japan.

***Abstract*. The questions of cosmic-ray energy spectrum known as 'knee' and 'ankle' are studied using recent results of the chemical composition measurements by direct observations and air-shower data. A formulation of the cosmic-ray energy spectrum above 10^{12} eV is made under the assumption of the cosmic-ray source distribution with various acceleration limits. From the comparison of the proposed model calculation with the experimental data, a new interpretation of the knee and the end of the galactic cosmic rays are discussed. In this model, the sharp knee is explained as being consisted of two components. The first component is a global component whose energy spectrum extends over wide range with gradual change of the power index by approximately 0.4 in $10^{14} - 10^{16}$ eV region. The second component is characterized by its heavy mass and hard spectrum of power index close to 2 with a cutoff at 4×10^{15} eV suggesting its origin as nearby source(s). Maximum energy of the galactic cosmic rays is estimated to be around 10^{18} eV for protons. The spectrum of the highest energy cosmic rays over 3 $\times 10^{18}$ eV apparently deviates from the proposed galactic cosmic-ray model indicating extragalactic origin.**

***Keywords*: Knee, Ankle, Cosmic-ray acceleration**

I. INTRODUCTION

It is well known that the energy spectrum of cosmic rays is well described by power law over a wide energy range covering more than 10 decades and it is regarded as the remarkable feature of non-thermal acceleration mechanism of the high energy cosmic rays. It is also known there is a sudden steepening of the power index from approximately -2.7 to -3.1 around 4×10^{15}eV exhibiting the shape of the spectrum like a 'knee'[1]. There has been a lot of works on the origin of the knee in terms of the acceleration mechanism[2], the propagation in the galaxy[3] or some nearby sources[4]. Another point of view is related to the nature of hadronic interactions[5]. In order to resolve the origin of the knee, the study of the chemical composition provides a key information, for the change of the energy spectrum can be related to the change of the chemical composition. A reasonable progress has been made recently on chemical composition study by direct observations on board balloon and air shower experiments at high mountain altitudes, where the shower development reaches nearly maximum at the knee energy range leading to a good energy estimation. On the other hand, development of high-energy gamma-ray astronomy provided evidences of electron acceleration at supernova remnant (SNR)[6] through diffusive shock acceleration (DSA) mechanism[7]. It is also known there are various sites of cosmic-ray acceleration including extragalactic ones as reviewed by Hillas[8]. The continuous energy spectrum of cosmic rays over the wide range suggests the universality of the acceleration mechanism of cosmic rays in these sources, namely, the DSA mechanism works at the every site of the cosmic-ray acceleration. The expected energy spectrum from such a source can be written by a power law formula multiplied by exponential cutoff factor as follows.

$$\frac{dJ}{dE} = \sum_z j_z E^{-\gamma_z} \exp[-\frac{E}{\varepsilon_z}], \tag{1}$$

where J denotes the flux of accelerated particles of the energy E, j_z the flux of the particles of atomic number z, γ_z the power index and ε_z is the acceleration-limit energy at a given source. Here, rigidity dependent cutoff is assumed as $\varepsilon_z = z \times \varepsilon_p$, where ε_p is the acceleration limit for protons. It is commonly discussed to interpret the knee as caused by acceleration limit of DSA mechanism at SNRs which is thought to be around 10^{15} eV. This scenario naturally leads to the steepening of the energy spectrum with increase of the average mass of cosmic rays for $E > \varepsilon_p$ and many observations around 10^{15} eV seem to support this tendency. However, this scenario works only at the vicinity of the knee energy region and one should introduce some other components such as very heavy element up to Uranium[1] and/or extragalactic particles to explain the all particle spectrum extending up to 10^{20}eV.

In this paper, a new framework of understanding the cosmic-ray energy spectrum extending over the wide range is proposed under the assumption of the universality of the DSA mechanism with various acceleration limits. The question of the maximum energy of the galactic cosmic rays is also discussed under proposed model through the comparison with recent results of the observation of the highest energy cosmic rays.

II. FORMULATION OF THE ENERGY SPECTRUM OF COSMIC RAYS

Experimentally, the energy spectrum of cosmic rays in wide energy range shows broken power law spectrum and it can be approximated by following formula except for the details of the knee structure which will be described later.

$$\frac{dj}{dE} = j_0 E^{-\gamma}(1 + E/\varepsilon_b)^{-\Delta\gamma}, \tag{2}$$

where ε_b denotes the break point, γ the power index in the energy range $E << \varepsilon_b$ and the power index at $E >> \varepsilon_b$ is expressed by $\gamma + \Delta\gamma$. Experimental value of $\Delta\gamma$ is known as 0.4 ± 0.1. Although this formula is frequently used in discussing the cosmic-ray spectrum, it is treated simply as empirical formula and it's physical meaning has not been understood very well. However, it is possible to derive eq.(2) from a simple assumption on the cosmic-ray accelerators.

Let $S(\varepsilon)d\varepsilon$ be the probability of the cosmic-ray accelerators with acceleration limit energy in $d\varepsilon$ at ε. Then the observed cosmic ray energy spectrum of a given chemical component is a superposition of the cutoff spectrum eq.(1) with weight $S(\varepsilon)$ and written as follows when we assume the chemical composition of the accelerated cosmic rays at each sources are the same in average.

$$\frac{dj}{dE} = \int_0^\infty j_0 E^{-\gamma} \exp(-\frac{E}{\varepsilon}) S(\varepsilon) d\varepsilon, \quad (3)$$

where $S(\varepsilon)$ is normalized as

$$\int_0^\infty S(\varepsilon) d\varepsilon = 1. \quad (4)$$

From eq.(2) and eq.(3), the function $S(\varepsilon)$ should satisfy,

$$\int_0^\infty \exp(-\frac{E}{\varepsilon}) S(\varepsilon) d\varepsilon = (1 + E/\varepsilon_b)^{-\Delta\gamma}. \quad (5)$$

One can find the solution of eq.(5) as,

$$S(\varepsilon)d\varepsilon = \frac{1}{\Gamma(\Delta\gamma)} (\frac{\varepsilon}{\varepsilon_b})^{-1-\Delta\gamma} \exp(-\frac{\varepsilon_b}{\varepsilon}) \frac{d\varepsilon}{\varepsilon_b}, \quad (6)$$

where Γ denotes the gamma function. Eq.(6) can be rewritten using a variable $x = \varepsilon/\varepsilon_b$ as

$$S(x)dx = \frac{1}{\Gamma(\Delta\gamma)} x^{-1-\Delta\gamma} \exp(-\frac{1}{x}) dx, \quad (7)$$

The shape of the function $S(x)$ is shown in Fig. 1 for $\Delta\gamma = 0.4$. Hence, the meaning of the break point ε_b can be understood as the acceleration limit energy by the minimum cosmic-ray accelerator, say type II SNR of the progenitor mass of 8 solar mass, and the probability of larger accelerators decreases as $\varepsilon^{-1-\Delta\gamma}$. The question of the maximum value of ε will be discussed in section III-B.

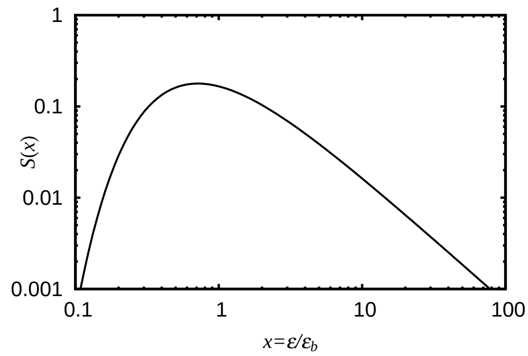

Fig. 1. Distribution of sources with acceleration limit ε ($\Delta\gamma = 0.4$). See the text for the definition of $\Delta\gamma$ and ε_b.

III. Comparison with experimental data

A. *Energy spectrum around the knee*

The energy range of the direct observations has been extended to 10^{14}eV region by recent long duration balloon flights at south pole and they provide the energy spectrum of protons and all elements of even atomic numbers up to irons. However, none of these direct observations claimed the break of the power law spectrum within their measured energy range. Therefore we made fitting of the experimental energy spectrum of each element with following power law formula without including the break point. The formula includes power index γ_z and absolute intensity j_z and empirically defined function $h(E)$ which describes the behavior at low energy region as explained by leaky box model[9]. Thus, the effect of the propagation of cosmic rays from the source to the earth is involved in these parameters.

$$\frac{dj}{dE} = j_z E^{-\gamma_z} h(E) \quad (8)$$

$$h(E) = 1 - \exp[-(\frac{E}{a_z})^{1.3}] \quad (9)$$

The numerical values of fit parameters can be found in a separate paper in this conference [10]. At higher energy range, Tibet air shower experiment [12] provides the proton spectrum in the energy range from 5×10^{14} to 10^{16} eV with steep power index of -3.1 ± 0.1 indicating the break of power law around several times 10^{14} eV. To fit the proton spectrum over wide energy range up to 10^{16} eV, broken power law formula (2) is used including the factor $h(E)$ as follows.

$$\frac{dj_z}{dE} = j_z E^{-\gamma_z} h(E) (1 + \frac{E}{z\varepsilon_b})^{-\Delta\gamma} \quad (10)$$

If one uses the formula (10) for each component and tries to obtain all-particle spectrum close to the air-shower data, $\varepsilon_b \simeq 7 \times 10^{14}$eV is the best parameter. Thus, we could describe the all-particle spectrum in wide range as the sum of the broken power law spectrum for each element, however, the data by Tibet data [11] around the knee has another structure than expected. Namely, the bending of the spectrum is more pronounced than the calculation as shown in Fig. 2, in which solid line represent present calculation (hereafter called global component). Remarkable feature in this figure is the excess of the spectrum in the energy range between 10^{14} eV and 10^{16} eV over the global component. Subtracting the global component from all-particle spectrum we can see a hard spectrum of power index close to 2, which is just the expected value of source spectrum before the modulation by the propagation, and it also shows cutoff feature indicating that the excess component is due to the contribution of the nearby source(s). The sharp cutoff of the excess component indicates that the

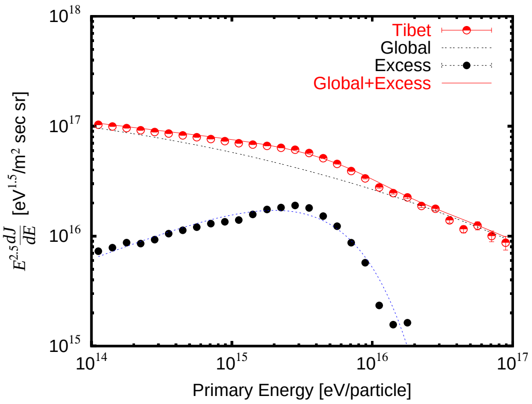

Fig. 2. All-particle spectrum around the knee. Solid line is the global component calculated by present model. Dashed line is the fit to the excess component seen in Tibet 3 data[11] which can be approximated as $f_x(E) \propto E^{-2} \exp(-E/4\text{PeV})$. The dot-chained line is the sum of the global component and $f_x(E)$

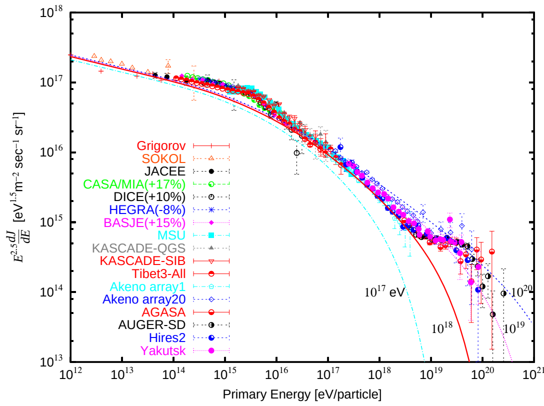

Fig. 3. All-particle spectrum in a wide energy range. The spectra around the knee are normalize to direct observations and the highest energy observations are normalize to Hires' result, keeping the relative values of correction factor the same as Berezinsky's method. Used correction factors are λ =0.72 and 0.6 for AGASA and YAKUTSK, respectively. AUGER is also plotted using $\lambda = 1.2$. Cited data: Grigorov[14], SOKOL[15], JACEE[16], Akeno[17], CASA/MIA[18], DICE[19], HEGRA[20], BASJE2004[21], MSU[22], KASCADE-QGS and KASCADE-SIB[23], Tibet3-All[11] Akeno array1[24], Akeno array20[24], AGASA[25], AUGER-SD[26], Hires2[27], Yakutsk[28]

chemical composition is not mixed as much as global component but its main component consists of elements with limited range of atomic numbers if we adopt the rigidity dependent acceleration limit. The result of Tibet data for proton and helium [12] shows that the excess component is heavier than helium. One possible interpretation of the excess component is to attribute its origin to type Ia SN, whose ejecta contains heavy elements of calcium and iron group abundantly, because the energy spectrum of the excess component can be well reproduced by assuming the chemical composition of the type Ia SN ejecta, which could be injected to some nearby acceleration site.

B. Energy spectrum in wide range

The maximum energy attained by DSA mechanism at SNRs is still under the theoretical and observational investigations. Recent works have revealed the possibility of much higher maximum energy than currently accepted values, say about 10^{14} eV by early theoretical works. As observed by Chandra and SUZAKU[6], nonlinear effect of the shock acceleration can increase the magnetic field at the SNR as strong as 100 μG which is higher by two order of magnitude than currently accepted value of a few μG.

In present work, the maximum energy ε_{max} is used as a parameter to calculate the all particle spectrum assuming that there is no energy dependence of the source chemical composition for the galactic cosmic rays. Here, the function $S(\varepsilon)$ is normalized as follows.

$$\int_0^{\varepsilon_{max}} S(\varepsilon) d\varepsilon = 1. \tag{11}$$

The formula (10) is then modified as follows.

$$\frac{dj_z}{dE} = \frac{\Gamma_2(\Delta\gamma, (E + z\varepsilon_b)/\varepsilon_{max})}{\Gamma(\Delta\gamma)} \times j_z E^{-\gamma_z} h(E) [1 + \frac{E}{z\varepsilon_b}]^{-\Delta\gamma}, \tag{12}$$

where $\Gamma_2(a, b)$ denotes the incomplete gamma function of the second kind. Experimental all particle spectrum in wide energy range is compared with the calculation by imposing maximum acceleration limit for protons as $\varepsilon_{p,max} = 10^{17}, 10^{18}, 10^{19}, 10^{20}$ eV (and $\varepsilon_{z,max} = z \times \varepsilon_{p,max}$) as shown in Fig. 3, in which following normalization of the experimental data are made for clearer comparison between the model calculation and experiments. The data around the knee are normalized to the direct observation data by shifting the energy scale within 20% as shown in parenthesis of the data title. The data of highest cosmic rays are normalized using Berezinsky's method. Berezinsky[13] pointed out there is a dip in the energy spectrum of the highest energy cosmic rays due to the energy loss by e^+e^- production process of extragalactic cosmic rays interacting with cosmic microwave background and proposed a method of the energy calibration using the position of the dip. If we apply this method by shifting the energies of AUGER, YAKUTSK and AGASA to meet with Hires' result, which is most smoothly connected with the spectrum at the lower energy range, the all particle spectrum in wide energy range can be shown as Fig. 3 where the deviation from the line of present calculation with $\varepsilon_{max} = 10^{18}$ eV is clearly seen around $\simeq 3 \times 10^{18}$ eV, which is currently called 'ankle'.

C. Average mass

Since present model assumes various acceleration sites in our galaxy with different acceleration limit but no change in the chemical composition at the source, average mass increases with primary energy due to the rigidity dependent cutoff except the knee region, where more complicated behavior can be expected because of the excess component. Average mass $< \ln A >$ is

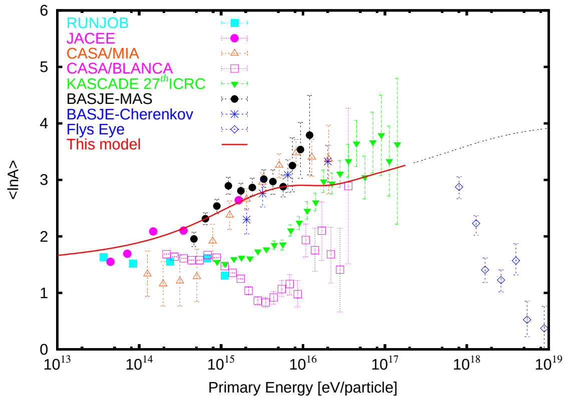

Fig. 4. Average mass. Cited data:RUNJOB[31], JACEE[16], CASA/MIA[18], CASA/BLANCA[32], KASCADE 27^{th}ICRC[33], BASJE-MAS and BASJE-Cherenkov[21], Fly's Eye[29]

calculated assuming that the excess component at the knee has the same chemical composition as the ejecta from type Ia SN. The calculated result for $< \ln A >$ is shown in Fig. 4 together with experimental data. Although the experimental data are still divergent because of the difficulty of the composition measurement at very high energy, some results agree with calculated line by present work.

There are two reports about the change of chemical composition at very high energy exceeding 10^{18} eV. Fly's eye[29] reported rapid decrease of the average mass which can be interpreted by contribution of proton component of the extragalactic origin and AUGER[30] reported in contrast the change of the elongation rate from that of light to heavy element and claims contribution of the extragalactic sources with mixed chemical composition.

IV. Conclusion and Discussion

Model of multiple sources with different acceleration limit and universality of the diffusive shock acceleration mechanism can describe galactic cosmic-ray spectrum up to 10^{18} eV. The gradual change of the power index of all-particle spectrum around 10^{15} eV which is known as the 'knee' is explained by the minimum acceleration limit denoted as ε_b in eq.(1). The numerical value of ε_b for proton can be determined from the break point of the proton spectrum and estimated as 7×10^{14} eV using available experimental data. The most possible sources around such energy are considered as type II SNRs and this value corresponds to the threshold value of the supernova explosion of the stellar mass around 8 solar mass. Observed knee has another structure showing the excess over the global component which makes the knee sharp and this excess can be attributed to nearby source(s).

The chemical composition of the excess component has the feature of the type Ia SN ejecta. The possible explanation of the excess component is that the ejecta of type Ia SN(s) are injected to the nearby acceleration site(s).

Acknowledgment

The author is grateful to the members of Tibet AS$_\gamma$ Collaboration for fruitful discussions and encouragement.

References

[1] Hörandel, J.R. 2003, Astropart. Phys., **19**, 193-220
[2] Berezhko, E. G. & Ksenofontov, L. G. 1999, JETP **89**, 391-403, Stanev, T., et al., 1993, Astron. Astrophys. **274**, 902-908, Jokipii, J.R. 1987, Ap. J. **313** 842-846, Jokipii, J.R. 1982, Ap. J. **255**, 716-720, Kobayakawa, K., et al. 2002, Phys. Rev. D, **66**, 083004, Völk, H.J. & Zirakashvili, V. N. 2004, Astron. Astrophys. **417**, 807-817
[3] Ptuskin, V.S., et al., 1993, Astron. Astrophys. **268**, 726-735, Candia, J., et al., 2002, JHEP **12**, 33
[4] Erlykin, A. D., & Wolfendale, A. W. 2005, Astropart. Phys., **23**, 1-9 Bednarek, W. & Protheroe, R. J., 2002, Astrop. Phys. **16**, 397-409
[5] Nikolsky, S. I. & Romachin, V. A. 2000, Phys. Atomic Nucl. **63**, 1799-1814
[6] Koyama et al. 1995, Nature, **378**, 255-258, Uchiyama, Y. et.al, 2007, Nature, **449**, 576-578, Aharonian, F.A. et al., 2004, Nature **432**, 75-77, Aharonian, F.A. et al., 2005, Science, **307**, 1938-1942
[7] Lagage, P.O & Cesarsky, C.J. 1983, A&A, **118**, 223-228
[8] Hillas, A.M., 1984, Ann. Rev. Astron. Astrophys. **22**, 425
[9] Gupta,M. & Webber, W.B. 1989, ApJ, **340**,1124-1134
[10] Amenomori, M. et al., 2009, 31st ICRC, HE2.1, icrc0294
[11] Amenomori, M et al, 2008, Astrophys. J, **678**, 1165-1179
[12] Amenomori, M et al, 2006, Phys. Lett. B **632**, 58-64
[13] Berezinsky, V, Gazizov, A and Grigorieva, S, 2006, Phys. Rev. D **74**, 043005
[14] Grigorov, N.L., et al. 1971, Proc. 12th Int. Cosmic Ray Conf. (Hobart), **5**, 1746
[15] Ivanenko, I.P. et al., 1993, Proc. 23rd ICRC, Calgary, **2**, 17
[16] Asakimori, K., et al. 1998, Astrophys. J., **502**, 278-283
[17] Nagano, M., Hara, T., Hatano, Y., Hayashida, N., Kawaguchi, S., Kamata, K., Kifune, T., Mizumoto, Y. 1984, J. Phys. G, **10**, 1295-1310
[18] Glasmacher, M. A. K., et al. 1999, Astropart. Phys. **10**, 291-302
[19] Swordy, S.P. et al. 2000, Astropart. Phys. **13**, 137-150
[20] Arqueros, F. et al. 2000, Astron. Astrophys. **359**, 682-694
[21] Ogio, S., 2004, Astrophys. J., **612**, 268-275
[22] Fomin, Y.A. et al. 1991, Proc. 22nd ICRC, Dubrin, **2**, 85
[23] Antoni et al. 2005, Astropart. Phys. **24**, 1-25
[24] Nagano, M., et al., 1992, J. Phys. G, **18**, 423-442
[25] Takeda, M., et al., 2003, Astropart. Phys. **19**, 447-462
[26] Yamamoto, T., [Pierre Auger Collaboration], 2007, Proc. 30th Int. Cosmic Ray Conf. (Merida). Roth, M., [Pierre Auger Collaboration], 2007, Proc. 30th Int. Cosmic Ray Conf. (Merida).
[27] Abbasi, R. U. et al. 2004, Phys. Rev. Lett. **92**, 151101-1-4
[28] Egorova,V.P. et al., 2004, Nucl. Phys. B (Proc. Suppl.), **136**, 3
[29] Bird, J. D. et al. 1993, Phys. Rev. Lett, **71**, 3401-3404
[30] Unger, M., 2007, 30th ICRC, Merida, arXiv:0706.1495v1 [astro-ph]
[31] Apanasenko, A.V., et al. 2001, Astropart. Phys. **16**, 13-46
[32] Fowler, J. W., et al., 2001, Astropart. Phys., **15**, 49-64
[33] Hörandel, J.R. 2001, Proc. 27th ICRC, Hamburg, **1**, 71