Magnetic field generation by a relativistic cosmic-ray ion beam in the precursor of parallel shocks

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***Abstract*. Magnetic-field generation by a relativistic ion beam propagating through an electron-ion plasma along a homogeneous magnetic field is investigated with 2.5D high-resolution particle-in-cell (PIC) simulations. The studies test recent predictions of a strong amplification of short-wavelength modes of magnetic turbulence upstream of nonrelativistic and relativistic parallel shocks associated with supernova remnants, AGN jets, and gamma-ray bursts. Representing the ion beam as a constant external current, i.e. excluding a backreaction of the magnetic turbulence on the beam, we observe non-resonant parallel modes with a wavelength and growth rate as predicted by analytic calculations. In this unrealistic setup the magnetic field is amplified to amplitudes far exceeding the homogeneous field, in agreement with recent MHD and PIC simulations. However, if all particles are fully modeled, the backreaction on the ion beam leads to filamentation of the ambient plasma and the beam, which in turn influences the properties of the magnetic turbulence. We compare the turbulence observed in our simulations with the dispersion relation for linear waves with arbitrary orientation of $\vec{k}$ and find good agreement. For mildly- and trans-relativistic beams, the instability saturates at field amplitudes comparable to the homogeneous magnetic field. This result matches our recent studies of nonrelativistically drifting hot cosmic-ray particles upstream of supernova-remnant shocks which indicated only a moderate magnetic-field amplification by nonresonant instabilities.**

***Keywords*: relativistic shocks, particle acceleration, magnetic fields**

I. Introduction

Particle acceleration at shocks invariably requires a continuous excitation of magnetic turbulence in the upstream region, which serves as a scattering medium to confine the energetic particles (cosmic rays) to the shock region for further acceleration. The cosmic rays as an ensemble drift relative to the upstream plasma, thus triggering a variety of instabilities that may lead to the growth of turbulent magnetic field. The distribution function of the cosmic rays is shaped by the scattering rate upstream, thus forcing a nonlinear relationship between the upstream plasma, the energetic particles, and small-scale electromagnetic fields.

While a full modeling of the upstream region is elusive to date, simulations of turbulence build-up using prescribed distribution functions for the upstream plasma and the cosmic rays can be invaluable tools for the study of the saturation processes and levels, as well as the backreaction of the evolved turbulence on the particles. In our recent work [1], [2] we have presented kinetic, Particle-in-Cell (PIC) simulations of cosmic rays with isotropic distribution, that slowly drift relative to the upstream plasma. Here we are interested in the interaction between the far upstream plasma and a cold relativistic beam of streaming particles. The situation is relevant far ahead of a SNR shock, where the cosmic rays will no longer have an isotropic distribution. Similar conditions, possibly involving a relativistic hot beam, might also be applied upstream of relativistic shocks of GRB and AGN.

It is known from studying non-relativistic beams in interplanetary space, that a competition between resonant and nonresonant modes arises, that exert a different backreaction on the beam [3]. For the case of a monoenergetic, unidirectional distribution of streaming cosmic rays, the growth rates for Alfvénic [4] and electrostatic [5] turbulence have been derived using quasilinear theory. Based on an analytical treatment, [6] found that also in this case nonresonant, purely growing modes may be expected to be significantly faster, although the growth rate falls off with the temperature of the background medium. We have performed a series of two-dimensional simulations for this setup to explore the relationship between this instability and that found for drifting cosmic rays, and to determine the impact of the backreaction of cosmic rays on the properties of the turbulence. We have explored the interaction of a cold, relativistic ion beam in the limit of a magnetized background plasma, for which the results of the analytical calculations of [6] apply.

II. Simulation setup

The code used in this study is a 2.5D (2D3V) version of the relativistic electromagnetic particle code TRISTAN with MPI-based parallelization ([7], [1]). In the simulations a cold, relativistic, and monoenergetic cosmic-ray ion beam with Lorentz factor γ_{CR} and number density N_{CR} streams along a homogeneous magnetic field $B_{\parallel 0}$ relative to the ambient electron-ion plasma. The ions of the ambient medium have a thermal distribution with number density N_i, in thermal equilibrium with the electrons. The electron population with density $N_e = N_i + N_{CR}$ contains the excess electrons required to provide charge-neutrality and drifts with $v_d = v_{CR}N_{CR}/N_e$ with respect to the background ions, so it provides a return current balancing the current carried by the ion beam. We have explored the system in the limit of a magnetized background plasma ($\omega \ll \Omega_i$), for which the results of the analytical calculations [6] apply. Specifically, we assumed $\gamma_{max}/\Omega_i = 0.2$, where

$$\gamma_{max} = \Im\omega \approx v_{CR}N_{CR}/2V_AN_i \quad (1)$$

is the growth rate of the most unstable nonresonant mode, Ω_i is the ion gyrofrequency, and $v_A = [B_{\parallel 0}^2/\mu_0(N_em_e+N_im_i)]^{1/2}$ is the plasma Alfvén velocity. For the super-Alfvénic flows, the relativistic cosmic-ray populations represent very dilute ion beams with density ratios $N_i/N_{CR} = 50$ and 125, respectively for two values of the Alfvén velocity considered in this study, $v_A = c/20$ and $v_A = c/50$. The simulations have been performed for the case of an ultrarelativistic beam with $\gamma_{CR} = 300$ and a slower beam with $\gamma_{CR} = 20$. We have also studied the case in which the beam is represented by a constant uniform external current, so that backreaction of the magnetic turbulence on the cosmic-ray beam is suppressed.

To assure the numerical accuracy of our simulations, we use a total of 16 particles per cell, and we apply the splitting method for the beam particles. The electron skindepth $\lambda_{se} = c/\omega_{pe} = 4\Delta$, where $\omega_{pe} = (N_ee^2/m_e\epsilon_0)^{1/2}$ is the electron plasma frequency and Δ is the grid cell size. We further assumed a reduced ion-electron mass ratio $m_i/m_e = 20$. This choice allows us to clearly separate the plasma and turbulence scales and yet use the computational box that can contain several wavelengths of the most unstable mode

$$\lambda_{max} \approx 2\pi(\gamma_{max}/\Omega_i)^{-1}\lambda_{si}, \quad (2)$$

where λ_{si} is the ion skindepth. We use grids with $(L_{\rm x}, L_{\rm y}) = (10.2\lambda_{max}, 7.4\lambda_{max})$, except for the cases in which a constant uniform external current is applied. In those runs $(L_{\rm x}, L_{\rm y}) = (7.4\lambda_{max}, 5.7\lambda_{max})$. Periodic boundary conditions are assumed for all boundaries.

III. Linear analysis

The growth rate and the wavelength of the most unstable purely-growing nonresonant mode given by equations 1 and 2 were obtained using linear kinetic analysis in the analytically tractable limit of a cold ambient plasma and for wavevectors $k_\parallel$ parallel to $B_{\parallel 0}$ ([10], [6]). We have numerically calculated the growth rates for arbitrary orientation of the wavevector, $\vec{k}$, in the zero-temperature limit. For a beam moving with $\gamma_{CR} = 20$ along a homogeneous magnetic field of strength given by the Alfvén velocity of $v_A = c/20$, the dominant unstable mode is the electrostatic Buneman mode between background ions and drifting electrons. The growth rate of this mode is about 10^3 times larger, and its wavelenght about 1.25×10^3 shorter than that of the nonresonant mode (1, 2). Our simulations do not resolve this mode. However, the Buneman instability is very sensitive to thermal effects and should saturate if the thermal velocity of ambient particles becomes comparable to their relative drift velocity. The initial electron thermal velocity in the simulations is thus set to values $v_{e,th} \lesssim v_d$ to ensure the quick saturation and dissipation of this unstable mode. Note, that such plasma parameters well reproduce the real conditions in astrophysical objects, and the Buneman mode will be relevant whenever the beam density is high, because v_d will be high as well in this case.

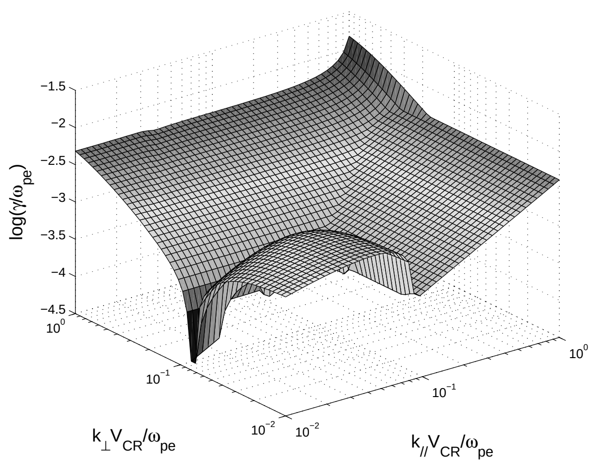

Fig. 1. Growth rate γ in units of the electron plasma frequency ω_{pe} in terms of the flow-aligned $k_\parallel$ and perpendicular $k_\perp$ wavectors for the parameters used in the simulation of the ion beam with $\gamma_{CR} = 20$ and plasma Alfvén velocity $v_A = c/20$. The spectrum is shown for the modes whose wavelengths are well contained in the simulation box.

In Figure 1 we show the growth rates in the reduced wavevector space $(Z_\parallel, Z_\perp)$, $Z_i = k_iV_{CR}/\omega_{pe}$, which is contained in our simulation box. The nonresonant mode is visible at $Z_\perp < 0.1$ and shows a broad peak centered at $Z_\parallel \approx 0.05$, corresponding to the estimate given by (2). However, it is not dominant even in the limited wavevector space covered in this study. In fact, the strongest growth occurs for $0.3 < Z_\perp < 1$, almost independent of $Z_\parallel$. The very peaked growth at $Z_\parallel \approx 1$ pertains to the Buneman instability between relativistic ion beam and ambient electrons. The growth at smaller $Z_\parallel$ represents the filamentation of the ambient plasma and the ion beam. The appearance of these fast-growing modes modifies the system, and one should expect that the properties of the nonresonant mode emerging in the nonlinear stage differ from those predicted in the analytical calculations. This is in fact what is observed in our simulations.

Note, that the growth rates as shown in Fig. 1 de-

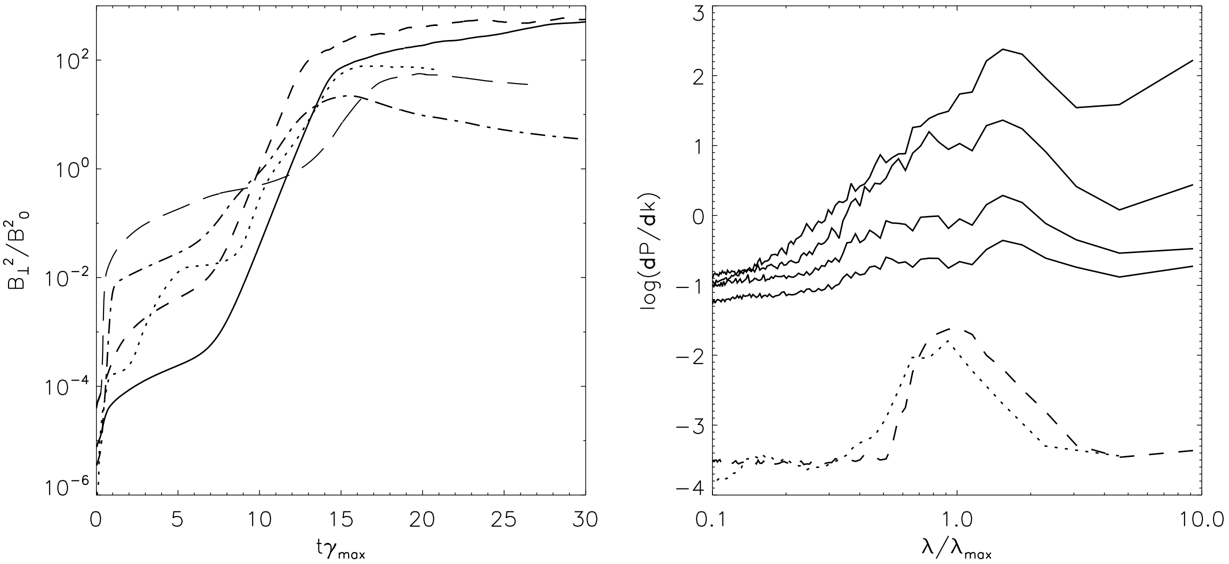

Fig. 2. Left panel: Temporal evolution of the energy density in the transverse magnetic field component, normalized to the homogeneous field strength. Time is dimensionless in units of the theoretically predicted inverse growth rate of the nonresonant mode, γ_{max}^{-1}. Simulations with constant cosmic-ray current are presented with the solid line for a plasma Alfvén velocity $v_A = c/20$, and the dashed line for $v_A = c/50$. The effects of cosmic-ray backreaction are shown for the case of $v_A = c/20$ and the beam Lorentz factor $\gamma_{CR} = 300$ (dotted) and $\gamma_{CR} = 20$ (dashdotted), and for $v_A = c/50$ and $\gamma_{CR} = 20$ (long dashed). Right panel: Fourier spectra of the perpendicular magnetic-field component B_z in wavelengths along the beam direction for the case with $v_A = c/20$. The parallel mode is expected at the wavelength λ_{max}. The dashed line shows results for a constant cosmic-ray current, and the dotted line for a faster beam with $\gamma_{CR} = 300$. Note the single peak structure centered on λ_{max}. The solid lines show the temporal evolution of the spectra for $\gamma_{CR} = 20$ during the linear stage (tγ_{max} = 6, 7, 9, 11, from bottom to top). The magnetic turbulence initially appears in a broad range of wavelengths.

pend on the parameters of the system under study. In particular, for ultrarelativistic beams the growth rate for the filamentation modes should be much smaller than for $\gamma_{CR} = 20$. Hence, for $\gamma_{CR} \gtrsim 100$ we primarily expect a competition between the nonresonant mode and the Buneman modes. If we replace the ion beam by an constant external current (which somewhat corresponds to $\gamma \gg 1$), then the ambient electrons do not interact with the ion beam, and a Buneman instability is not excited. The evolution of the system is then artificially dominated by the nonresonant instability [8].

IV. Simulation results

The temporal evolution of the energy density in the transverse magnetic field component normalized to the homogeneous field strength is shown in Fig. 2 (left panel). If the backreaction on the cosmic rays is suppressed, i.e. a constant uniform external current is applied, then a purely-growing parallel mode of magnetic turbulence appears in the plasma. Its growth rate ($\gamma \approx 0.8\gamma_{max}$ for the two cases with $v_A = c/20$ and $c/50$) and wavelength (dashed line in the right panel of Fig.2) agree well with those predicted by quasi-linear analytical calculations. The mode represents a purely magnetic, circularly polarized, and aperiodic transverse wave. The interactions of the magnetic turbulence with the plasma are related to the return current carried by the ambient electrons, $\vec{j}_{ret}$. In the later stage the $\vec{j}_{ret} \times \delta B_\perp$ force induces motions and turbulence in the background plasma, subsequently causing the turbulence to turn nearly isotropic and highly nonlinear. As in the case of drifting cosmic rays [1] and nonrelativistic beams [3], the saturation of the magnetic-field growth proceeds via bulk acceleration and occurs when the bulk velocity of the background plasma approaches the cosmic-ray ion beam speed. The amplitude of the field perturbations is larger for smaller Alfvén velocity, in agreement with [9]. For the case with $v_A = c/50$, the magnetic-field amplitude reaches $\delta B_\perp/B_{\parallel 0} \simeq 25$, which is close to the maximum obtained with MHD simulations ([10], [11], [12], see also [8]), in which the cosmic rays are also represented by a constant current. We will now describe the behavior of the system including the response of the relativistic cosmic-ray ion beam.

If the cosmic rays are treated fully kinetically, the dynamics of the system changes. The interaction of the ion beam with the plasma quickly leads to plasma and beam filamentation which is modified by a Buneman instability between ion beam and plasma electrons. The Buneman beam-electron interactions produce mainly electrostatic, slightly oblique turbulence whose wavelength of $\sim 25\Delta$ parallel to the direction of the beam is in very good agreement with the predictions of our linear analysis (section III). The mode grows very fast, causing density fluctuations in the beam and electron plasma. However, in the simulations its amplitude quickly saturates and is subsequently kept at a moderate level. The Buneman mode dissipates only after filamentation and nonresonant modes have strongly backreacted on the ion beam in the nonlinear stage.

The properties of the magnetic turbulence depend on the Lorentz factor of the beam. For an ultrarelativistic beam with $\gamma_{CR} = 300$ (dotted lines in Fig. 2), the filamentation is weak and the parallel nonresonant mode appears with the theoretically predicted wavelength. Its growth rate is initially $\gamma \approx 0.94\gamma_{max}$ and decreases during the nonlinear evolution. As one can see in Fig. 2 (left panel), the peak amplitude of the magnetic field perturbations, $\delta B_\perp/B_{\|0} \simeq 9$, is close to that obtained with constant external current (solid line) at the onset of the saturation of the turbulence growth ($t \sim 15\gamma_{max}^{-1}$). It appears that in this phase the high beam Lorentz factor provides sufficient stiffness to the ion beam that its backreaction is suppressed, rendering the system response similar to that for a constant external current. The similarity ends when the saturation kicks in, though. The subsequent dissipation of the turbulence in the run with $\gamma_{CR} = 300$ is much stronger than in the case of a constant external current, which places in doubt the accuracy of simulations that use a constant external current to describe the highly nonlinear phases in the evolution of the system.

Results for a system with a mildly-relativistic beam with $\gamma_{CR} = 20$, and for $v_A = c/20$ and $c/50$, are presented in Fig. 2 (left panel) with dash-dotted and long-dashed lines, respectively. As our linear analysis of section III shows, the filamentation modes at perpendicular wavevectors $k_\perp \approx \lambda_{se}$ are strong in this case. They cause filamentation in the ambient plasma and the ion beam, before the nonresonant parallel modes have emerged. As one can see in Fig. 2, these modes do not lead to magnetic-field perturbations of significant amplitude. Nevertheless, their action on the ambient plasma changes its properties, which considerably influences the characteristics of the purely-growing parallel modes. The nonresonant modes appear in a broad range of wavelengths around λ_{max} (Fig. 2, right panel), and the growth rate of the magnetic-field perturbations is only $\sim 0.4\gamma_{max}$. The backreaction of the turbulence on the system further enhances the filamentation in the beam and the plasma, and leads to the saturation and dissipation of the magnetic turbulence at a level comparable with the homogeneous magnetic field. The peak amplitudes for the two cases with $v_A = c/20$ and $c/50$ are $\delta B_\perp/B_{\|0} \simeq 4.5$ and $\delta B_\perp/B_{\|0} \simeq 7.5$, respectively, showing once again that instabilities operating in a less-magnetized medium provide a stronger field amplification.

V. Conclusion

We have studied the interaction of a cold, relativistic ion beam penetrating a cold plasma composed of electrons and ions. We have presented 2.5D PIC simulations, complemented with a linear analysis of the dispersion relation for linear waves with arbitrary orientation of $\vec{k}$, for parameters that permit the growth of nonresonant, purely-magnetic parallel modes [6]. Our research is relevant for the understanding of the structure of, and particle acceleration at shocks in SNR, GRB, and AGN, for which radiation modeling suggest that the magnetic field near the shock is strongly amplified.

We observe a close competition of the nonresonant mode with the filamentation instability and Buneman modes, in close correspondence with the linear dispersion relation. The specific choice of parameters determines which of the three modes of instability dominates. In some cases filamentation is initially important and modifies the later evolution of the parallel nonresonant mode. In all cases we find that a representation of the ion beam by a constant current, as is routinely done in MHD studies, is suboptimal, because it suppresses part of the nonlinear response of the system, delays the saturation processes, and leads to a significant overestimate of the magnetic-field amplitude in the later stages of the evolution.

As in the case of drifting cosmic rays [1], [2] and nonrelativistic beams [3], the saturation of the magnetic field growth proceeds via bulk acceleration. For mildly- and trans-relativistic beams, the instability saturates at field amplitudes comparable to the homogeneous magnetic field. These results match our recent studies of nonrelativistically drifting hot cosmic-ray particles upstream of SNR shocks which also indicated only a moderate magnetic-field amplification by nonresonant instabilities.

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