Kinetic simulations of turbulent magnetic-field growth by streaming cosmic rays

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***Abstract*. The acceleration of cosmic rays to high energies is thought to occur in supernova remnants via the mechanism of diffusive shock acceleration. Efficient acceleration requires turbulent, amplified magnetic fields in the shock's upstream region. We present results of multidimensional particle-in-cell simulations aimed at observing the magnetic field amplification that is expected to arise from the cosmic-ray current ahead of the shock. We find that the initial structure and peak strength of the amplified field is somewhat sensitive to the choice of parameters, but that the field growth saturates in a similar manner in all cases: the back-reaction on the cosmic rays leads to net transfer of momentum to the interstellar medium, substantially weakening their relative drift while also implying the development of a modified shock.**

***Keywords*: Supernova remnants, particle acceleration, magnetic fields**

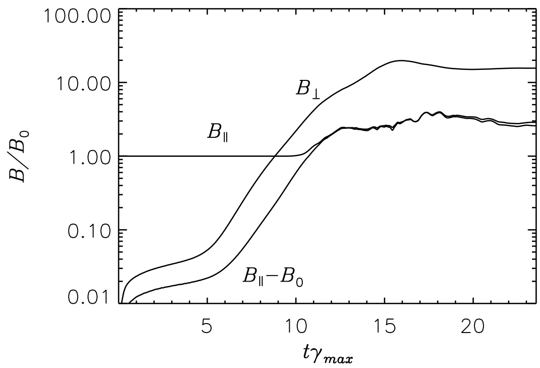


Fig. 1. Evolution of the magnetic field components displaying the growth and saturation of the instability predicted in [3]. The magnetic field amplitude is in units of the initial homogeneous field, and time is in dimensionless units set by the calculated inverse growth rate γ_{max}^{-1} from that instability.

I. Introduction

The forward shocks of young shell-type supernova remnants may be efficient acceleration sites for Galactic cosmic rays. The theory of diffusive shock acceleration (DSA) provides a promising mechanism by which particles can attain nonthermal energies in such an environment, provided they are confined to the shock vicinity; for review, consult [1] and references therein. The interstellar magnetic field alone appears to be incapable of providing the confinement necessary for cosmic rays to reach "knee" energies via the DSA mechanism. The need for a much stronger, turbulent magnetic field has attracted significant attention to the question of upstream magnetic field amplification.

It is well known that ion beams can generate magnetic turbulence via resonant and non-resonant interactions [2]. Bell [3] suggested that non-resonant magnetic field amplification may also be caused by the cosmic-ray current expected in the cosmic-ray precursor of a quasi-parallel shock. While substantial field amplification is seen in magnetohydrodynamical (MHD) simulations that assume a constant cosmic-ray current, the approximations of MHD may not be appropriate for modeling the non-linear evolution and eventual saturation of this instability; we must use kinetic methods to simulate these later stages with accuracy, including any back-reaction on the cosmic rays. Our earlier work [4] (hereafter N08) used 3-D particle-in-cell (PIC) simulations to model the growth and saturation of a similar current-driven instability. We relaxed the condition assumed in the calculations of B04 that the growth rate must be much less than the ion gyrofrequency, and we found that an oblique filamentary mode dominated the initial magnetic field growth. Riquelme and Spitkovsky [5] confirmed the findings of N08, and in particular investigated the parameters for the transition between the filamentation mode seen in N08 and the parallel-wave mode seen in the MHD simulations, and verified that the parallel mode appears only in the regime with $\omega \ll \Omega_i$. However, despite the difference in initial unstable modes, the non-linear characteristics of the system observed in both works [4], [5] were similar. In particular, both works observed only modest amplification of the magnetic field, and saturation by the same mechanism.

Here, we report more recent work, in which we simulate the current-driven instability and its saturation in a parameter regime in which the non-relativistic ion gyrofrequency far exceeds the predicted growth rate of the current-driven instability.

II. Simulation setup

To study the growth and saturation of magnetic turbulence in response to a cosmic-ray current, we use a 2.5D (2D3V) version of the relativistic electromagnetic particle-in-cell code TRISTAN with MPI-based parallelization. We model the cosmic rays (number density N_{CR}) as an isotropic, mono-energetic (Lorentz

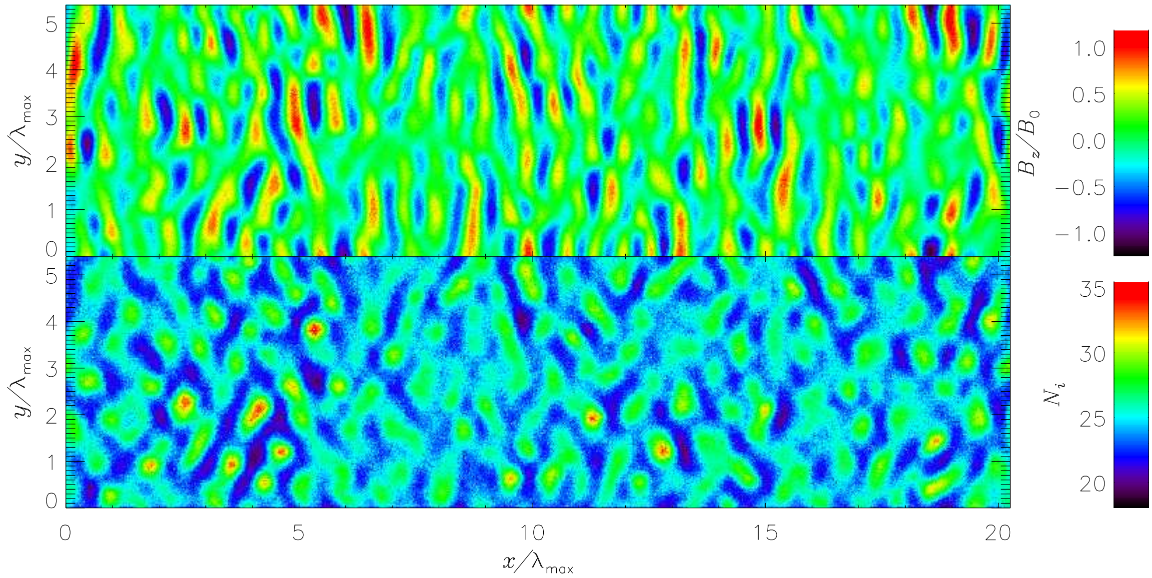


Fig. 2. The perpendicular magnetic field (top) and density of plasma ions (bottom) at $t\gamma_{max} = 7.0$. The parallel-mode structure is clearly visible in the magnetic field, and localized density fluctuations about the mean (25 ions per cell) are small in amplitude.

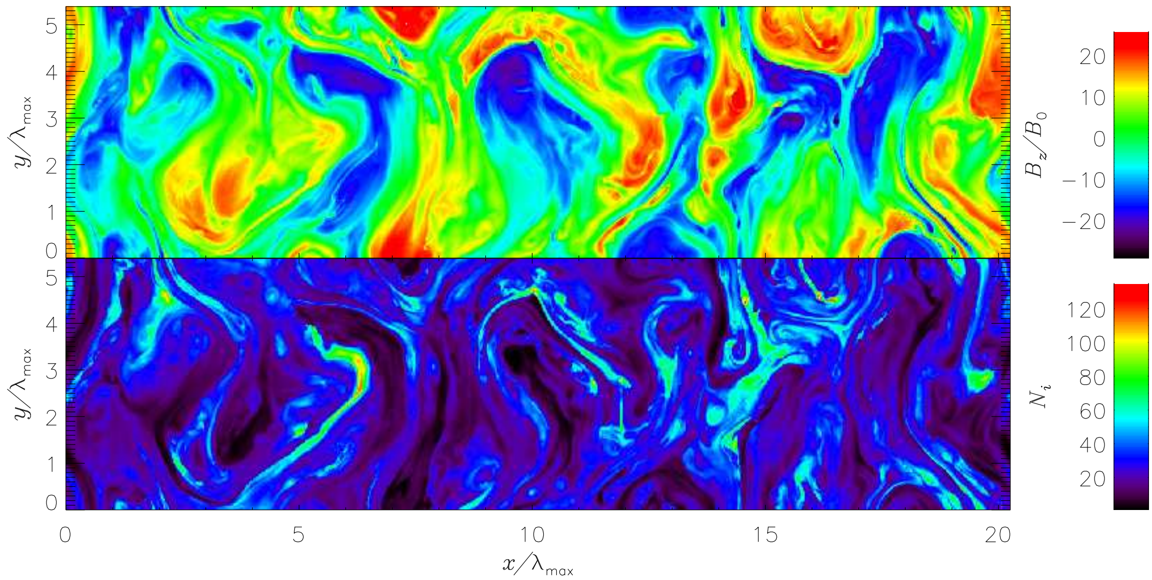


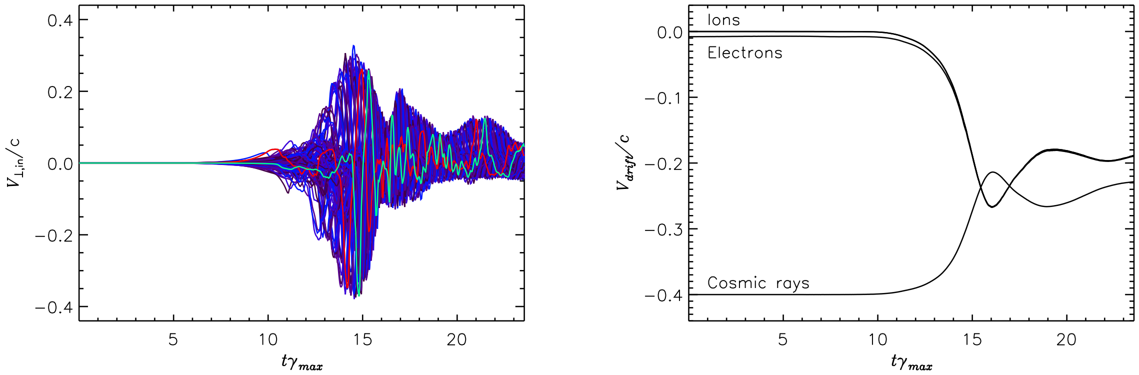
Fig. 3. The perpendicular magnetic field (top) and density of plasma ions (bottom) at $t\gamma_{max} = 14.0$. The plasma density fluctuations are strong and well correlated with the magnetic field structures during the non-linear stage of magnetic field growth. The structures move rapidly on oblique trajectories at this time in the simulation (see Figures 4a and 4b).

factor γ_{CR}) population drifting at the shock velocity v_{sh} along a homogeneous magnetic field $B_{\|0}$ relative to an electron-ion plasma. The plasma ions, number density N_i and mass m_i, have a thermal distribution in equilibrium with the electrons, $N_e = N_i + N_{CR}$, which drift at velocity $v_d = v_{sh} N_{CR}/N_e$ relative to the ions to provide a return current that balances the current carried by the cosmic rays. For such a configuration, B04 predicts an instability in which the most unstable mode, circularly polarized fluctuations with wavenumber $k_{\|max}$, grows at a rate $\gamma_{max} = \Im\omega = v_A k_{\|max}$, where

$$k_{\|max} = \frac{e N_{CR} v_{sh} B_{\|0}}{2 N_i m_i v_A^2} \tag{1}$$

and $v_A = \left[B_{\|0}^2/\mu_0 \left(N_e m_e + N_i m_i\right)\right]^{1/2}$ is the Alfvén velocity of the plasma. This result is subject to the restriction that the wave mode satisfy $\omega \ll \Omega_i$; that is, that the growth rate of the instability must be much less than the nonrelativistic ion gyrofrequency.

In the simulations presented here, $m_i/m_e = N_i/N_{CR} = 50$. We set $B_{\|_0}$ such that $v_A = 0.01c$. The cosmic rays drift at $v_{sh} = 0.4c$, have Lorentz factors $\gamma_{CR} = 50$, and we use electron skin depth $\lambda_{se} = c/\omega_{pe} = 4\Delta$ for the main production run, where $\omega_{pe} = (N_e e^2/m_e \epsilon_0)^{1/2}$ is the electron plasma frequency and Δ is the grid cell size. The initial ion particle density is 25 per cell, and to improve statistics



(a) Transverse plasma velocity

(b) Drift velocity

Fig. 4. Left: The transverse in-plane velocity of the plasma in regions of size $(0.45\lambda_{max})^2$. Two regions have been selected and superimposed in lighter colors for clarity. Right: Momentum transfer from the cosmic rays to the background plasma removes their relative drift, limiting the amplification of the magnetic field. The "overshoot" at $t\gamma_{max} \sim 16$ is an artifact of the smaller computational grid used in this run and is not present in our production run.

we split each cosmic-ray ion into ten simulated particles. We use a computational grid that includes several $\lambda_{max} \equiv 2\pi/k_{\|max}$ in the cosmic-ray drift direction, as well as a minimum of $2\lambda_{max}$ in the transverse direction to allow adequate room for spatial variations. Periodic boundary conditions are applied on all boundaries.

Our primary simulation's computational grid was $9000\Delta \times 2400\Delta$, admitting $20.26\lambda_{max} \times 5.40\lambda_{max}$. An additional supplementary run with finer spatial resolution, $\lambda_{se} = 6\Delta$, confirmed that the evolution of the system is not strongly affected by artificial numerical effects.

III. Results and discussion

Our simulations reproduce the initial growth of a non-resonant streaming instability, but it becomes compressive while $\delta B \ll B_{\|0}$. As the turbulent magnetic field becomes comparable in strength to the homogeneous field, the plasma begins to move and the cosmic rays slow down, reducing the current and saturating the magnetic-field amplification at a level of 10–20 $B_{\|0}$.

Figure 1 shows the evolution of the components of the magnetic field parallel and perpendicular to the cosmic-ray current. After a few growth periods γ_{max}^{-1} in which small random fluctuations bring about a gradual amplification of the field, a perturbation $B_\perp$ arises that undergoes amplification by the non-resonant instability. The amplitude increases exponentially at a rate initially near γ_{max}, but within a few growth periods $\delta B \sim B$ and the rate decreases. The growing field $B_\perp$ has the structure of a stationary wave, circularly polarized and oriented parallel to the drift direction. The turbulent contribution to $B_\|$ also grows but remains roughly an order of magnitude smaller than $B_\perp$ during the linear phase.

Even before the turbulent field surpasses the homogeneous background field, the evolution starts to differ significantly from a purely growing parallel mode. As seen in Figure 3, the dominant length scale of the fluctuations grows and significantly exceeds λ_{max}. Substantial variation appears along the direction transverse to the drift, reaching the maximum length scale allowed by the periodic boundary conditions, even on our largest computational grid.

The deceleration of the field growth accompanies the onset of turbulent plasma motion. Parcels of ambient plasma whose size is comparable to λ_{max} move and collide, giving rise to considerable density fluctuations. Figure 4a indicates that the plasma parcels can attain a considerable range of speeds in the directions transverse to the cosmic-ray drift. The strong perpendicular field and turbulent plasma allow bulk momentum transfer from the cosmic rays to the ambient plasma, so superimposed on the turbulent motions, the plasma begins to drift more and more rapidly in the direction of the cosmic rays, while the cosmic rays simultaneously slow down (Figure 4b). In our simulations, the speeds converge such that the relative drift is roughly an order of magnitude smaller than its initial value. In addition to slowing down in bulk, the instantaneous rest-frame distribution of cosmic rays is modified, as shown in Figure 5. Some anisotropy is introduced, which may have consequences for the modeling both of radiation and particle acceleration.

It is important to note that the ability of our simulations to accurately portray the evolution of the magnetic field and the plasma properties at very late times, beyond the saturation, is limited. In addition to the limitation imposed by the finite size of the simulation domain, the assumptions of environmental homogeneity become increasingly unsound. Changes in the plasma flow speed imply density adjustments in order to conserve mass flux. Also, as the shock approaches, spatial gradients in the cosmic-ray precursor and ambient plasma properties play an increasingly important role. Furthermore, the time in which upstream amplification can occur

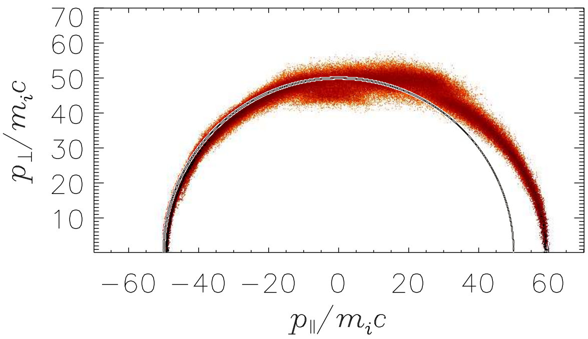


Fig. 5. The phase-space distribution of cosmic rays in the instantaneous rest frame of the population at time $t\gamma_{max} \approx 15.7$, compared with the initial distribution (thin semicircle).

before the system is overtaken by the shock is only $\sim 40\gamma_{max}^{-1}$ for our parameters, assuming efficient Bohm-type diffusion so that the shock sweep-up timescale is $c r_{CRg}/3v_{sh}^2$ (where r_{CRg} is the cosmic-ray gyroradius) [4]. Finally, the interactions between the shock itself and any small-scale spatial variations not only in plasma density but also in velocity may have a non-negligible impact on the injection process and the resulting distribution of energetic particles [6], [7].

However, the reduction of the cosmic-ray current (by the converging drift velocities of their population and the ambient plasma) serves to underscore the need to treat these particles kinetically. The back-reaction from the turbulent plasma and amplified magnetic field is non-negligible, with the most prominent consequence being saturation of the field growth at a lower amplitude than might be expected from a constant cosmic-ray current.

Finally, it is worth noting that although the initial dominant mode of magnetic-field amplification agrees with the calculations of B04, the later evolution and saturation mechanism have much in common with the simulations performed in N08, in which the choice of simulation parameters exposed an oblique filamentary mode as the dominant instability. It seems it doesn't matter what form the initial linear instability takes.

IV. Summary and Conclusion

We have simulated the turbulent amplification of the interstellar magnetic field upstream of a nonrelativistic shock using a kinetic 2.5-dimensional particle-in-cell code. Our observations are that the non-resonant streaming instability [2], [3] is seen for our choice of parameters, but that the plasma quickly evolves and saps the bulk momentum from the drifting cosmic rays, effectively removing their current and saturating the magnetic-field amplification to $\sim 20B_{\parallel 0}$. This saturation mechanism is independent of the initial linear instability, occurring as a result of the back-reaction on the cosmic rays. Strong transverse plasma motions arise in conjunction with the turbulent magnetic field exceeding the homogeneous background field, and the plasma begins to drift, suggestive of a cosmic ray-modified shock, until its speed approximately matches that of the cosmic-ray population. The evolution past this point is beyond the reach of our current simulation method, as it requires additional input to reflect the changing external environment.

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