Accelerated ions and selfexcited Alfvén waves at the interplanetary shock

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***Abstract*. We study the diffuse energetic ion April 5, 1979, event and related Alfvén waves upstream at the quasi-parallel interplanetary shock within the quasilinear approach. Time-dependent diffusive transport equation for accelerated ions (protons and α-particles) and wave transport equation are numerically solved in order to obtain the spectra of ions and Alfvén waves and their spatial distributions. It is shown that the power law spectrum of accelerated ions extends almost up to 1 MeV. Alfvén waves excited by accelerated ions are confined within the frequency range $(10^{-2}-1)$ Hz and their spectral peak with the wave amplitude comparable to the interplanetary magnetic field value corresponds to the frequency $(2-3)\times10^{-2}$ Hz. High frequency part of wave spectrum undergoes absorption by thermal protons. Calculated accelerated ion spectra and associated Alfvén wave spectra are compared with the experiment.**

***Keywords*: particle diffusive shock acceleration, Alfvén waves generation, interplanetary shocks**

I. INTRODUCTION

Numerous measurements accomplished in interplanetary space demonstrate that intensive acceleration processes take place in the vicinity of interplanetary shock fronts. Some aspects of energetic particle generation and the associated generation of Alfvén waves can be understand in the frame of simplified plane-wave approach of the diffusive shock acceleration theory [1].

At the same time the geometrical factors (finite, increase shock size, adiabatic cooling in the expanding solar wind) essentially influences the acceleration [2], [3]. These factors determine the maximal energy of accelerated particles and its evolution during the shock propagation.

We study here the diffuse energetic ion April 5, 1979, event and related Alfvén waves upstream at the quasi-parallel interplanetary shock within the quasilinear approach.

II. MODEL

The front of the interplanetary shock has a complicated nonspherical form. One can expect that the most effective acceleration takes place at the front part of the shock, where the shock velocity is the highest and the interplanetary magnetic field (IMF) has a small angle with the shock normal in the inner heliosphere, at heliocentric distances $r\le R_E=1$ AU. This part of the shock will be considered as a part of the sphere of radius R_S which increases in time with the the speed $V_S=dR_S/dt$. As far as the transverse size of the acceleration region $L_\perp$ is large enough ($L_\perp\sim R_S$), and fast particles are strongly magnetized ($\kappa_\parallel\gg\kappa_\perp$), the spherical approximation can be used. In this case, the diffusive transport equation for the particle distribution function $f(r,v,t)$ in the upstream region $r>R_S$ has a form

$$\frac{\partial f}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\kappa\frac{\partial f}{\partial r}\right)-w_n'\frac{\partial f}{\partial r}-\frac{f}{\tau_\perp}+ +\left[\frac{1}{3r^2}\frac{\partial(r^2w_r')}{\partial r}\right]v\frac{\partial f}{\partial v}\ , \qquad (1)$$

where v is the ion velocity and t is the time of their acceleration, $\kappa=\kappa_\parallel\cos^2\psi+\kappa_\perp\sin^2\psi$ is the ion diffusion coefficient, describing their diffusion along the shock normal, $\kappa_\parallel$ and $\kappa_\perp$ are the parallel and perpendicular relative to interplanetary magnetic field (IMF) $\boldsymbol{B}$ ion diffusion coefficients, ψ is the angle between IMF $\boldsymbol{B}$ and radial direction, w_r' and w_n' are scattering center speed components along the radial direction and along the shock normal direction respectively.

The diffusion of particles is due to their resonant interaction with Alfvén waves. It is described by the diffusion coefficients

$$\kappa_\parallel=\frac{v^2B^2}{32\pi^2\omega_BE_w\left(k=\rho_B^{-1}\right)}\ ,\ \kappa_\parallel\kappa_\perp=\rho_B^2v^2/3\ , \qquad (2)$$

where $\rho_B=v/\omega_B$ is the gyroradius, $\omega_B=ZeB/(Amc)$ is the gyrofrequency, m and e are the proton mass and charge, c is the speed of light, Z and A are the ion charge and mass numbers, and $E_w(k)=d(\delta B^2/8\pi)/d\ln k$ is the Alfvén wave energy density per logarithm of the wave number k.

The third term on the right-hand side of Eq. (1) effectively describes the particle escape from the acceleration region through transverse diffusion with the time scale $\tau_\perp=L_\perp^2/\kappa_\perp$. Following [2] we assume that $L_\perp=0.6R_S$.

Since in our model Alfvén waves play a role of scattering centers, their speed in the local plasma frame is directed along the IMF and it has a value

$$c_c=c_a(E_w^+-E_w^-)/E_w\ , \qquad (3)$$

where $c_a = B/\sqrt{4\pi\rho}$ is the Alfvén velocity, ρ is the solar wind density. Therefore the scattering center speed components, contained in Eq. (1), can be represented in the form

$$w'_r = w + c_c \cos\chi \ , \qquad (4)$$

$$w'_n = w \cos\varphi + c_c \cos\psi \ , \qquad (5)$$

where χ is the angle between IMF $\boldsymbol{B}$ and $\boldsymbol{w}$ and φ is the angle between the solar wind speed $\boldsymbol{w}$ and shock normal.

We neglected the shock modification by the pressure of accelerated particles. Therefore the shock front is treated as discontinuity at which the medium speed relative to the shock front $u_1 = V - w\cos\varphi$ undergo a jump from the value u_1 at $r = R_S + 0$ to $u_2 = u_1/\sigma$ at $r = R_S - 0$, where

$$\sigma = 4\big/\left(1 + 3/M_1^2\right) \qquad (6)$$

is the shock compression ratio, $M_1 = u_1/c_{s1}$ is the Mach number, $c_s = \sqrt{\gamma_g k_B T/m}$ is the sound speed, $\gamma_g = 5/3$, T is the proton temperature and k_B is the Boltzmann constant. The subscript 1 (2) marks the quantities corresponding to the point immediately upstream (downstream) of the shock front.

We suppose that accelerated particle distribution just behind the shock front is close to uniform. Therefore the boundary condition for particle distribution function at the shock front can be written in the form

$$\frac{u'_1 - u'_2}{3} v \frac{\partial f}{\partial v} = \kappa_\parallel \cos^2\psi \frac{\partial f}{\partial r} + Q_0 \ , \qquad (7)$$

where

$$u' = V - w'_n \qquad (8)$$

is the scattering center speed along the shock normal,

$$Q_0 = u_1 \frac{N_{inj}}{4\pi v_{inj}^2} \delta(v - v_{inj}) H(R_S - R_0) \qquad (9)$$

is the source concentrated at the shock front that provides the injection into acceleration of $N_{inj} = \eta N_1$ particles from each unit volume intersecting the shock front, η is injection rate, v_{inj} is the speed of the injected particles. We use in our calculation $v_{inj} = 4c_{s2}$ where $c_{s2} = u_1\sqrt{\gamma_g(\sigma - 1) + \sigma/M_1^2}\Big/\sigma$ is the downstream sound speed .

Since the shock front is the only source where the particles are injected into acceleration, the problem should be solved for the initial and boundary conditions

$$f(r, v, t = t_0) = 0 \ \ , \ \ f(r = \infty, v, t) = 0 \ \ . \qquad (10)$$

The spectrum of Alfvén waves $E_w = E_w^+ + E_w^-$ is a mixture of waves propagating along (against) the radial direction relative to the the solar wind with energy density E_w^+ and E_w^- respectively.

We use equal amount of oppositely propagating Alfvén waves $E_{w0}^+ = E_{w0}^-$, that is consistent with the measurements at the Earth's orbit [4], [5].

The Alfvén wave spectrum E_{w0} is modified near the shock due to waves generation and damping by accelerated particles and due to their damping on thermal protons according to the wave transport equation

$$\frac{\partial E_w^\pm}{\partial t} + u_1^\pm \frac{\partial E_w^\pm}{\partial r} = \mp 2\Gamma E_w^\pm - L \ , \qquad (11)$$

where

$$\Gamma(k) = \frac{32\pi^3 c_a}{kc^2 v^2} \sum_s \frac{(Ze)^2}{Am} \kappa_\parallel \left(v = \frac{\omega_B}{k}\right) \times$$

$$\times \int_{v_{\min}}^{\infty} dv v^3 \left(1 - \frac{\omega_B^2}{k^2 v^2}\right) \cos\psi \frac{\partial f}{\partial x} \qquad (12)$$

is the wave growth (damping) rate due to accelerated particles distribution [6]; $v_{\min} = \max(v_{inj}, \omega_B/k)$; "s" refers the type of ion (for the simplicity we omit this index in the corresponding quantities); L is the wave damping rate. Note that the expression (12) for the wave growth rate has more appropriate form compared with what was initially derived by Lee [1]. The flow speed in Eq. (11) in the shock frame along the radial direction in upstream region is determined by the expression

$$u_1^\pm = V - w\cos\varphi \pm c_a \cos\psi \ . \qquad (13)$$

According to the experiment [7] the background Alfvén wave frequency spectrum

$$E_{w0}(\nu) = E_{w0}(k = 2\pi\nu/w\cos\chi)/\nu \qquad (14)$$

at the Earth's orbit has a power-law form $E_{w0}(\nu) \propto \nu^{-\lambda}$ with a break at $\nu_b \sim 0.1$ Hz so that spectral index value is $\lambda \approx 3/2$ at $\nu < \nu_b$ and $\lambda \approx 3$ at $\nu > \nu_b$. Here $\nu = kw\cos\chi/2\pi$ is the frequency seen by a stationary observer. The break frequency $\nu_b = k_b w/2\pi$ can be determined from the relation [7]

$$k_b = \omega_B / \left(c_a + v_T\right) \ , \qquad (15)$$

where $v_T = \sqrt{k_B T/m}$ is the thermal proton speed.

The break in the Alfvén wave spectrum is due to their damping on thermal protons. It is described by the damping term in the wave transport equation (11):

$$L = \Gamma_p (E_w^\pm - E_{w0}^\pm) \ , \qquad (16)$$

where

$$\Gamma_p(k) = m_1 \omega_B \left[\left(\frac{kc}{\omega_p}\right)^2\right]^{m_2} \exp\left[-4m_3^2 \left(\frac{\omega_p}{kc}\right)^2\right], \qquad (17)$$

$\omega_p = \sqrt{4\pi N e^2/m}$ is the proton plasma frequency, $m_1 = 0.66\beta^{0.43}$, $m_2 = 1.17 + 0.4\beta^{0.4}$, $m_3 = 0.31/\beta^{0.26}$, and $\beta = 8\pi k_B N T/B^2$ is the plasma parameter [8].

The formulated problem (1) – (17) is solved numerically.

III. Results and discussion

We apply our model to describe the diffuse energetic ion event and related Alfvén waves upstream of the interplanetary shock, measured at R_E by ISEE 3 satellite on April 5, 1979, event (see [9], [10], [11]).

Since IMF is nearly radial for the considered case we use the radial dependence of its strength $B = B_E(R_E/r)^2$, where $B_E = B(r = R_E) = 13.5\times10^{-5}$ G [11].

The solar wind proton number density are described by the same kind of dependence $N = N_E(R_E/r)^2$. According to the experiment proton number density is $N_E = 4.4$ cm^{-3} [11]. We take into account the acceleration of α-particles with their number density in the solar wind $N_\alpha = 0.05N$.

We adopt the radial dependence of the proton temperature $T = T_E(R_E/r)^{0.93}$ according to the experiment [12] and its value at the Earth's orbit $T_E = 3.5\times10^6$ K. The value of solar wind speed for the considered event is $w = 410$ km/s [11].

We take the background Alfvén wave energy density in the form

$$E^{\pm}_{w0}(r,k) = E^{\pm}_0(k/k_0)^{-\beta}(r/R_E)^{-\delta}\ , \qquad (18)$$

where $k_0 = \omega_{B_E}/v_{inj}$. We use suitable values of $E_0 = E^+_0 + E^-_0 = 1.5\times10^{-13}$ erg/cm^3, $\beta = 0.15$ and $\delta = 4$; $E^+_0 = E^-_0$ (see [4], [5], [13]).

The injection rate $\eta = 2.5\times10^{-3}$ of acceleration particles is needed to fit the data.

We consider the propagation of the shock through the inner heliosphere in range distances $0.2 \le R_S/R_E \le 1$. Following [14] we adopt the dependence of the shock speed radial component in the form $V_S = V_0(R_0/r)^{0.27}$, where $V_0 = 1000$ km/s, $R_0 = 0.2R_E$ and $V_S(R_E) = 647$ km/s [11].

We use the values of relevant angles $\psi_1 = 44°$, $\psi_2 = 70°$, $\varphi = 53°$ and $\cos\chi = 0.84$ determined in the experiment [11].

We present in Fig. 1 the proton energy spectra

$$J(\varepsilon) = v^2 f(v,t)/m\ , \qquad (19)$$

calculated for two different distances from the shock front together with the experimental data, measured during two different periods of time. It is seen that proton spectrum at the shock front very well fits the measurements accomplished just behind the shock front during the period 01:30-01:40 UT. Note that satellite intersected the shock front at 01:21 UT.

However calculated spatial upstream particle distribution is considerably steeper compared with the experiment. It is seen in Fig. 1 where we present also the proton spectrum calculated for the distance from the shock $d = 8.5\times10^{-4}$ R_E and the measurements accomplished during corresponding period of time 00:00-00:10 UT ($d \approx 1.8\times10^{-2}$ R_E). One can see that calculated spectrum goes progressively below the experiment as the energy decreases. This is presumably due to the fact that calculated proton diffusion coefficient is smaller than the actual one.

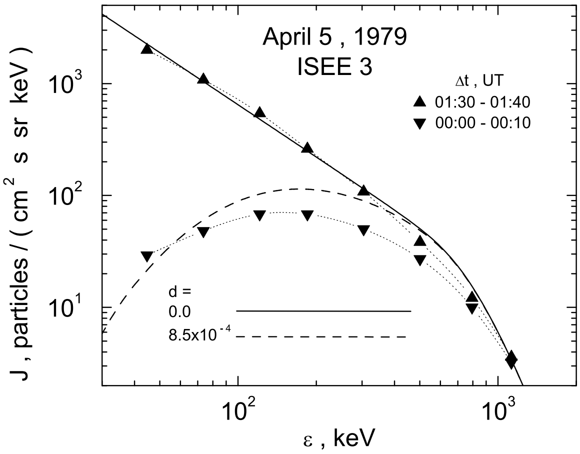

Fig. 1: The energy spectrum of accelerated protons, calculated for two different distances upstream of the shock together with the experimental data, measured by ISEE 3 satellite for April 5, 1979, event [9] at corresponding two periods of time.

This is confirmed by Fig. 2 where calculated Alfvén wave spectrum together with the experimental data are presented. It is seen that calculated wave spectrum at the shock front ($d = 0$) exceeds the experientially determined spectrum by a factor of about four within the relevant frequency range $\nu = 10^{-2} - 10^{-1}$ Hz.

Note that protons with energies $\varepsilon > 10$ keV are scattered by waves with frequencies $\nu \le \nu_{inj}$, where $\nu_{inj} = \omega_B u/(2\pi v_{inj}) \approx 0.1$ Hz. The low-frequency part of the wave spectrum ($\nu < \nu_{inj}$) reflects the shape of the accelerated particle spectrum. The wave damping on thermal ions, which is described by the source term L in the wave transport equation, leads to the steepening of the wave spectrum above the break frequency $\nu_b \approx 0.5$ Hz. Note that calculated and measured Alfvén wave spectra have a similar shape at relevant frequencies $\nu = 10^{-2} - 10^{-1}$ Hz.

The pressure of accelerated particles

$$P = \frac{4}{3}\pi\sum_s Am\int\limits_{v_{inj}}^{\infty} dvv^4 f_s(v)\ , \qquad (20)$$

at shock front ($r = R_S$) reach the value $P = 0.13\rho u^2$ at the Earth's orbit, $R_S = R_E$. It is small compared with the ram pressure ρu^2, that justifies the neglect of the shock modification by the particle backreaction.

The overall energy densities of the Alfvén waves excited by accelerated particles

$$W = \int\limits_0^{\infty}[E_w(x=0,\nu) - E_{w0}(\nu)]d\nu\ , \qquad (21)$$

at the shock front ($R_S = R_E$) reach the value $W = 0.2E_B$ at $R_S = R_E$, where $E_B = B^2/8\pi$ is the IMF energy density.

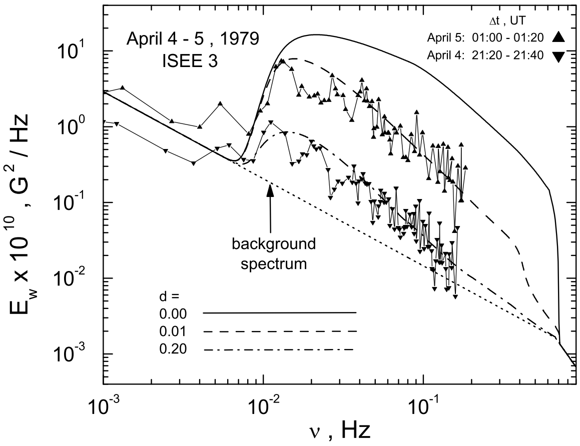

Fig. 2: The Alfvén wave spectrum as a function of frequency, calculated for two distances upstream of the shock together with the experimental data measured by ISEE 3 satellite for April 5, 1979, event [10] at corresponding two periods of time.

Note that in the case of the intense Alfvén wave generation by the shock accelerated particles the expected overall energy densities of the Alfvén waves W is given by the expression [15], [6]

$$E_w(R) = W/(B_1^2/8\pi) = (u_1 \cos\psi/c_a)\big(P/\rho_1 u_1^2\big). \quad (22)$$

It gives the value which agrees within 10% with numerically calculated value. It is therefore unclear why calculated Alfvén wave spectrum considerably exceeds the experimental one.

IV. Summary

Quasilinear model, developed to describe the diffuse energetic ion events and related Alfvén waves upstream of the interplanetary shock, was applied to April 5, 1979, event detected with instruments on ISEE 3 satellite. The relation between the theory and experiment is rather controversial. Calculated energy spectrum of accelerated protons at the shock front is very well consistent with the experiment, whereas calculated spectrum of associated Alfvén waves considerably exceeds experimentally determined spectrum. The reason of this discrepancy is unclear at the moment.

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