Nonlinear Cosmic Ray Propagation and Confinement in the Galaxy

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***Abstract*. Nonlinear particle diffusion theories are employed for achieving a more realistic description of cosmic ray propagation in the Galaxy. We demonstrate theoretically that the energy dependence of the particle mean free path along the mean magnetic field is directly proportional to $E^{0.6}$. We also consider transport of ultrahigh-energy cosmic rays. Usually it is assumed that particles having a Larmor radius larger than the largest scale of the interstellar turbulence cannot be confined to the Galaxy. We demonstrate that this limit does not exist in reality and is a consequence of the quasilinear approximation used in previous investigations.**

***Keywords*: Cosmic Rays, Turbulence, Galaxy**

I. Introduction

Radio synchrotron radiation intensity and polarisation surveys of our own and external galaxies (for a review see Sofue *et al.* 1986) have revealed that the interstellar medium is transversed by a large-scale ordered magnetic field ($\vec{B}_0 = B_0\vec{e}_z$) with superposed magnetic turbulence ($\delta\vec{B}$). The turbulence field has a broad spectrum of scales with the largest one being $10 - 100$pc (see, e.g., Beck 2007, and references therein). Charged particles (cosmic rays) in such turbulent plasmas are scattered and accelerated through resonant and non-resonant interactions with stochastic electromagnetic fields.

It is assumed that the most important effect for the propagation and the confinement of cosmic rays is diffusion along the mean magnetic field. This process is described by the parallel diffusion coefficient $\kappa_\parallel$ or the parallel mean free path $\lambda_\parallel = 3\kappa_\parallel/v$. These transport parameters also control the cosmic ray anisotropy A (see Schlickeiser 1989)

$$A \approx \frac{\lambda_\parallel}{3g}. \tag{1}$$

Here we have used the characteristic spatial gradient of the cosmic ray density g. Cosmic ray gradients derived from diffusive galactic GeV gamma-ray emissivities (Strong & Mattox 1996) suggest a value of $g \approx 2$ kpc.

In the following (for more details see Shalchi & Schlickeiser 2005 and Shalchi et al. 2009) we explore the propagation and the confinement of high energetic cosmic rays in the interstellar medium (ISM). Previous investigations of interstellar particle transport are based on quasilinear theory (QLT, see Schlickeiser 2002 for a review). In the present article this standard description is replaced by nonlinear transport theories. Such nonlinear theories have already sucessfully been applied to the transport of charged particles in the solar system (e.g., Shalchi *et al.* 2006).

II. Alternative Explanation of the Abundance Ratio of Secondary to Primary Galactic Cosmic Rays

A. Parallel mean free paths in the ISM

The observed ratio of secondary to primary cosmic ray nuclei indicates that primary cosmic rays at relativistic energies penetrate a total column density of matter $X = n_0\tau v$ during their residence time τ in the Galaxy, where n_0 is the average density of the interstellar gas and v is the cosmic ray velocity. For diffusive propagation the mean residence time τ can be expressed by the system size (thickness of Galactic disk) L and the parallel spatial diffusion coefficient $\kappa_\parallel$ as $\tau = L^2/\kappa_\parallel$, so that

$$X(R) = \frac{n_0 v L^2}{\kappa_\parallel(R)} = \frac{3n_0L^2}{\lambda_\parallel(R)} \propto \lambda_\parallel^{-1} \tag{2}$$

where we used the parallel mean free path $\lambda_\parallel = 3\kappa_\parallel/v$. The measured decrease of the abundance ratio of secondary to primary cosmic ray nuclei as B/C and N/O at kinetic energies above 1 GeV/nucleon, implies a variation of the total column density as a function of rigidity R as (Swordy et al. 1990)

$$X(R) = 6.9\,(R/[\text{20GV/nucleon}])^{-a} \text{ g cm}^{-2} \tag{3}$$

with

$$a = 0.6 \pm 0.1. \tag{4}$$

The rigidity dependence and therefore the parameter a is controlled by the rigidity dependence of the inverse parallel mean free path $X \propto \lambda_\parallel^{-1}$ and, therefore,

$$\lambda_\parallel \propto R^{0.6\pm0.1}. \tag{5}$$

Here we have used the parameter $R = R_L/l_{slab}$ ($R_L = v/\Omega = pc/(|q|\,B_0)$ is the Larmor radius, l_{slab} the slab bendover scale) which is a measure for the particle momentum.

In plain diffusion or leaky-box transport models without distributed stochastic acceleration the implied value of $a = 0.6$ is not in accord with the prediction of the quasilinear theory (e.g. Schlickeiser & Miller 1998, Schlickeiser 2002) with a Kolmogorov power spectrum of magnetic fluctuations $g(k) \propto k^{-5/3}$:

$$\lambda_\parallel^{QLT} \propto R^{1/3} \tag{6}$$

For this reason models with distributed stochastic acceleration have been favored (e.g. Jones et al. 2001) despite the disagreement of the implied weak energy dependence of the secondary-to-primary ratios at energies ≥ 20 GeV/nucleon with the high-energy HEAO-3 data by Binns et al. (1981) on the sub-Fe/Fe ratio.

B. The Weakly Nonlinear Theory

Recent test particle simulations have shown that nonlinear effects are important if the parallel mean free path is calculated (see Shalchi et al. 2004). Therefore, a nonlinear theory for cosmic ray transport was derived to describe particle transport in agreement with simulations. The WNLT (weakly nonlinear theory) of Shalchi et al. (2004) is based on the quasilinear formulation but nonlinear effects were included. By the formal replacement

$$\pi\delta\left(k_\parallel v_\parallel + n\Omega\right) \to \frac{D_\perp k_\perp^2 + \omega}{\left(D_\perp k_\perp^2 + \omega\right)^2 + \left(k_\parallel v_\parallel + n\Omega\right)^2} \tag{7}$$

with

$$\omega = \begin{cases} \frac{2D_{\mu\mu}}{1-\mu^2} & \text{for perpendicular diffusion} \\ 0 & \text{for pitch-angle diffusion.} \end{cases} \tag{8}$$

we can substitute the sharp delta function of quasilinear theory by the Breit-Wigner resonance function of WNLT. Resonance-broadening in WNLT arises from pitch-angle diffusion, described by the Fokker-Planck coefficient $D_{\mu\mu}$ and perpendicular diffusion, described by the Fokker-Planck coefficient $D_\perp$. Thus, we have a coupled system of nonlinear Fokker-Planck coefficients within WNLT: $D_{\mu\mu} = D_{\mu\mu}(D_{\mu\mu}, D_\perp)$, $D_\perp = D_\perp(D_{\mu\mu}, D_\perp)$. The formulas for these two Fokker-Planck coefficients are given in Shalchi et al. (2004). This system can be solved numerically and the parallel mean free path can be calculated by applying the relation (see, e.g., Earl 1974)

$$\lambda_\parallel = \frac{3\kappa_\parallel}{v} = \frac{3v}{8}\int_{-1}^{+1} d\mu \, \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)} \tag{9}$$

with the pitch-angle cosine $\mu = v_\parallel / v$ and the particle velocity v.

In Shalchi & Schlickeiser (2005) we have calculated the parameter $D_{\mu\mu}(\mu)$ by using the weakly nonlinear theory (WNLT, Shalchi et al. 2004) for the well-established slab/2D composite model. Then we have employed Eq. (9) to compute the parallel mean free path. The results are shown and discussed in the next section.

C. Explanation of the secondary-to-primary ratio

Fig. 1 shows the parallel mean free path $\lambda_\parallel / l_{slab}$ as a function of the dimensionless rigidity $R = R_L / l_{slab}$ calculated with WNLT in comparison with QLT results. For our calculation we used the parameters which should be appropriate for the interstellar medium: $s = 5/3$, $l_{slab} = 2 \cdot 10^{18}$cm, $l_{2D} = 0.1 \cdot l_{slab}$, $B_0 = 0.4$nT, $\delta B_{slab}^2 / \delta B_{2D}^2 = 0.25$, $\delta B^2 / B_0^2 = 0.5$; As explained in Shalchi & Schlickeiser (2005) the results of the current article are only valid for kinetic energies satisfying $E_0 < E_{kin} < 2.4 \cdot 10^6$GeV. According to Fig. 1 we find indeed $\lambda_\parallel \sim R^{0.6}$. The value $a = 0.6 \pm 0.1$ in Eq. (3) can by explained theoretically by the WNLT, which thus provides evidence for the presence of nonlinear transport of Galactic cosmic rays.

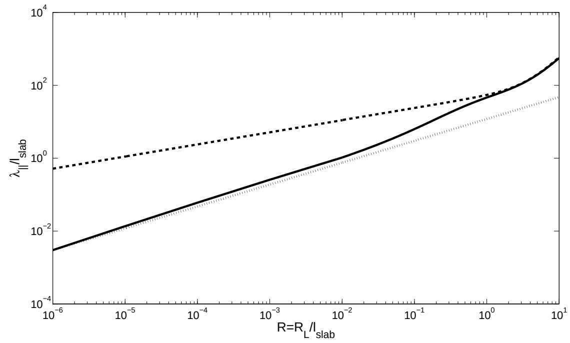

Fig. 1. The parallel mean free path devided by the slab-bendover scale $\lambda_\parallel / l_{slab}$ as a function of $R = R_L / l_{slab}$ (R_L =Larmor-radius, l_{slab} =slab bendover scale). We have shown WNLT results (solid line), QLT results (dashed line), and a $R^{0.6}$-fit (dotted line).

III. Transport of ultrahigh-energy cosmic rays

A. Quasilinear theory and the Hillas limit

The confinement and the anisotropy of cosmic rays is controlled by the parallel spatial diffusion coefficient $\kappa_\parallel$, which is related to the parallel mean free path via $\lambda_\parallel$. According to Eq. (9), these two parameters can be expressed as integrals over the inverse pitch-angle Fokker-Planck coefficient $D_{\mu\mu}$. The latter parameter can easily be computed by using QLT. A further assumption which is often used is that the stochastic magnetic fields can be replaced by the so-called magnetostatic slab model for which we assume $\delta B_i(\vec{x}) = \delta B_i(z)$. The combination of quasilinear theory and the magnetostatic slab model is also known as standard quasilinear theory. In this case we find (see, e.g., Teufel & Schlickeiser 2002 for a detailed derivation of this formula)

$$D_{\mu\mu} = \frac{2\pi v^2 (1-\mu^2)}{B_0^2 R_L^2} \int_0^\infty dk_\parallel \, g^{slab}(k_\parallel) \times \left[R_+(k_\parallel) + R_-(k_\parallel)\right] \tag{10}$$

with the quasilinear resonance function

$$R_\pm(k_\parallel) = \pi\delta\left(v\mu k_\parallel \pm \Omega\right) \tag{11}$$

and the turbulence wave spectrum $g^{slab}(k_\parallel)$. The particles experience only interaction with a certain wavenumber fulfilling the resonance condition $v\left|\mu k_\parallel\right| = \Omega$ corresponding to $|\mu| R_L \left|k_\parallel\right| = 1$. This scattering condition is known as gyroresonance. Real physical systems have a certain size and, therefore, it is reasonable to introduce a largest turbulence scale with $g^{slab}(k_\parallel < L_\parallel^{-1}) = 0$. As a consequence we find $D_{\mu\mu}(|\mu| > L_\parallel / R_L) = 0$ and for ultrahigh particle energies ($R_L > L_\|$) we obtain by using Eq. (9)

$$\lambda_\|(R_L > L_\|) = \infty. \tag{12}$$

Thus, ultrahigh energetic particles with a Larmor radius larger than the scale $L_\|$ cannot be confined to the Galaxy. This limit

$$R_{L,H} = \frac{p_H c}{|q|\, B_0} = L_\| \tag{13}$$

is known as the Hillas limit (Hillas 1984). For relativistic particles ($E = cp$) we have for the characteristic energy until which particles can be confined to the Galaxy

$$E_H = |q|\, B_0 L_\|. \tag{14}$$

For protons and by assuming $B_0 = 0.6$nT for the galactic magnetic field we have $|q|\, B_0 = 0.18$eV/m $= 5.4 \cdot 10^{15}$eV/pc. By using that the largest scale of turbulence is $10 - 100$pc (see Beck 2007), Eq. (14) becomes

$$E_H = 5.4 \cdot 10^{16}\text{eV} - 5.4 \cdot 10^{17}\text{eV}. \tag{15}$$

Particles with higher energies cannot be scattered and are therefore not confined to the Galaxy. For energies larger than E_H we have $\lambda_\|(E > E_H) = \infty$ and, therefore, we find for the cosmic ray anisotropy (see Eq. (1)) $A(E > E_H) = \infty$. Therefore, the turbulent magnetic fields can only isotropize cosmic rays up to $5.4 \cdot 10^{16}$eV $-5.4 \cdot 10^{17}$eV.

B. *Nonlinear transport in slab turbulence*

So far we have shown systematically how the Hillas limit can be derived. Mathematically the Hillas limit is a consequence of the sharp gyroresonance function of Eq. (11). The first assumption leading to this resonance function is that the stochastic magnetic fields can be approximated by magnetostatic slab fluctuations $\delta B_i(\vec{x}, t) = \delta B_i(z)$. Modifications of this model and the consequences for the confinement and anisotropy of ultrahigh-energy cosmic rays were recently discussed (see Vukcevic & Schlickeiser 2007). The second assumption is that the real particle motion can be approximated by the unperturbed motion corresponding to the application of quasilinear theory. Nonlinear theories for cosmic ray scattering along the mean magnetic field were developed previously (see, e.g., Völk 1973, Jones *et al.* 1973, Owens 1974, Shalchi *et al.* 2004, Shalchi 2005, Shalchi 2009).

In the first assumption one has to distinguish between the turbulence geometry (slab or isotropic turbulence) and dynamical effects (e.g., plasma wave propagation effects). The latter effect is usually coupled to particle transport parameters via the ratio v_A/v (Alfvén speed devided by the particle speed). Nonlinear effects are coupled via $\delta B/B_0$ (strength of the stochastic or wave field devided by the mean magnetic field). Since we are interested in high energetic particles we have $v_A/v \ll 1$. Observations indicate (see Beck 2007) that in the interstellar medium $\delta B/B_0$ is of order unity. Therefore, it is reasonable to assume that nonlinear effects are more important than dynamical turbulence and wave propagation effects.

In this section we employ the second order quasilinear theory (SOQLT, Shalchi 2005) in combination with the magnetostatic slab model (calculations for isotropic turbulence are presented below). In the second order theory we no longer assume unperturbed orbits. Instead, quasilinear theory is used in order to compute improved orbits. Mathematically, the second order approach leads to a modified (broadend) resonance function (see Shalchi 2005 for a detailed derivation)

$$R_\pm^{SOQLT}(k_\|) = \frac{\sqrt{\pi} B_0}{v k_\| \delta B} \exp\left[-\frac{(v\mu k_\| \pm \Omega)^2 B_0^2}{(v k_\| \delta B)^2} \right]. \tag{16}$$

The quasilinear resonance function (see Eq. (11)) can be recovered by considering the limit $(v k_\| \delta B/B_0)^2 \to 0$.

For the wave spectrum we assume the arbitrary form introduced in Shalchi & Weinhorst (2009) in the context of turbulence studies

$$\begin{aligned} g^{slab}(k_\| \le L_\|^{-1}) &= \frac{D(s,q)}{2\pi} \delta B^2 l_{slab} \\ &\times \frac{|k_\| l_{slab}|^q}{\left[1 + (k_\| l_{slab})^2\right]^{(s+q)/2}}. \end{aligned} \tag{17}$$

Here we have used the normalization constant $D(s,q) = \Gamma\left[(s+q)/2\right] / \left\{2\Gamma\left[(s-1)/2\right]\Gamma\left[(q+1)/2\right]\right\}$, the bendover scale of the turbulence l_{slab}, the energy range spectral index q, and the inertial range spectral index s. The spectrum is correctly normalized for $s > 1$, $q > -1$, and $l_{slab} \ll L_\|$.

It is straightforward to combine Eqs. (9), (10), (16), and (17) to compute the parallel mean free path as a function of the particle energy (the mathematical details can be found in Shalchi 2005). In Fig. 2 we show the proton parallel mean free path for $l_{slab} = 1$pc, $q = 0$, $s = 5/3$ (corresponding to a standard Kolmogorov inertial range), and $L_\| = 100$pc for different values of $\delta B/B_0$. For very weak turbulence ($\delta B/B_0 = 0.1$) we find a rapid increase of the parallel mean free path at $R_L = L_\|$ similar to the quasilinear result. For intermediate strong turbulence ($\delta B/B_0 = 0.3$) and strong turbulence ($\delta B/B_0 = 1$), however, we only find a weak effect. Cut-off effects in the wave spectrum are less important except for $\delta B/B_0 \ll 1$. The rapid increase at $R_L = L_\|$ obtained by quasilinear theory is the consequence of the assumption of unperturbed orbits. Such idealized orbits do not exist in reality due to the permanent interaction between the particles and the stochastic magnetic fields.

C. *Nonlinear transport in isotropic turbulence*

Above we have combined the second order quasilinear theory with the slab model. This model, however, could be invalid for the interstellar medium. Tautz *et*

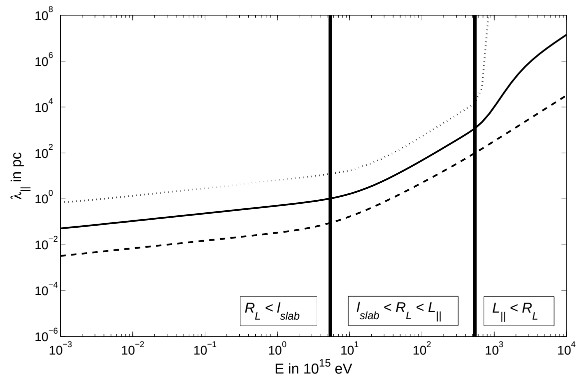

Fig. 2. The parallel mean free path for protons and slab turbulence versus the particle energy for $\delta B^2/B_0^2 = 0.01$ (dotted line), $\delta B^2/B_0^2 = 0.1$ (solid line), and $\delta B^2/B_0^2 = 1$ (dashed line). All results were obtained by employing second order quasilinear theory. Standard quasilinear theory yields $\lambda_\parallel = \infty$ for $R_L > L_\parallel$.

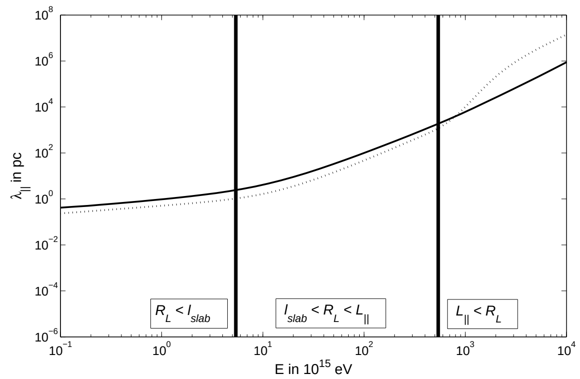

Fig. 3. The parallel mean free path for protons versus the particle energy for $\delta B/B_0 = 0.3$ and $q = 0$. Shown are the results for slab turbulence (dotted line) and isotropic turbulence (solid line). All results were obtained by employing second order quasilinear theory. Standard quasilinear theory yields $\lambda_\parallel = \infty$ for $R_L > L_\parallel$ and slab turbulence and $\lambda_\parallel = \infty$ for all values of R_L in the case of isotropic turbulence.

al. (2008) have applied the second order theory to isotropic turbulence. In Fig. 3 the second order results are shown for the parallel mean free path in slab and isotropic turbulence. For the turbulence parameters we have assumed $\delta B/B_0 = 0.3$ and $q = 0$. The results are nearly in coincidence. It seems that the turbulence geometry has a very weak influence on the parallel mean free path, the confinement, and the anisotropy of galactic cosmic rays.

IV. Summary

We applied the recently proposed weakly nonlinear theory (WNLT) of cosmic ray transport to examine the rigidity dependence of the parallel mean free path. This is interesting and important because the rigidity dependence controls the abundance ratio of secondary to primary cosmic ray nuclei as B/C and N/O at kinetic energies above 1 GeV / nucleon. Within quasilinear theory (QLT) with the rigidity dependence $\lambda_\parallel \propto R^{1/3}$ models with distributed stochastic acceleration have been favored. We have demonstrated that WNLT provides a different rigidity dependence of the parallel mean free path, namely $\lambda_\parallel \propto R^{0.6}$. Therefore we come to the conclusion that within a nonlinear transport theory the abundance ratio of secondary to primary cosmic ray nuclei can be explained, which thus provides evidence for the presence of nonlinear transport of galactic cosmic rays.

Furthermore we have investigated transport of ultrahigh-energy cosmic rays. It is a well-known effect in cosmic ray physics that particles only experience parallel diffusion for $R_L < L_\parallel$. For higher particle energies the motion is scatter-free (ballistic) and, therefore, such particles cannot be confined to the Galaxy. Furthermore we find a vanishing cosmic ray anisotropy for $R_L > L_\parallel$ corresponding to $E > E_H = 5.4 \cdot 10^{17}$eV for $L_\parallel = 100$pc. This limit is a consequence of the sharp gyroresonant interaction arising due to the application of quasilinear theory. We have applied the second-order quasilinear theory developed by Shalchi (2005) to the transport of ultrahigh energetic cosmic rays. As demonstrated we no longer find an infinitely large parallel mean free path for $R_L > L_\parallel$ due to the nonlinear resonance broadening effect, which is taken into account in the second order approach. For slab as well as for isotropic turbulence we find a finite parallel mean free path and, therefore, a finite cosmic ray anisotropy above E_H.

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