Studying shower to shower fluctuations with simulations

Patricia M. Hansen*, **Ricardo A. Vazquez**†, **Jaime Alvarez-Muñiz**†

* *Dep. de Física, IFLP CONICET Facultad de Ciencias Exactas-Univ. Nacional de La Plata C.C. 67, 1900, Argentina*
†*Depto. de Física de Partículas & Instituto Galego de Física de Altas Enerxías, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain.*

***Abstract*. The study of shower to shower fluctuations is hampered by the thinning algorithm, which introduces artificial fluctuations and distorts the probability distribution. We consider the effect of the thinning in the calculation of the shower to shower fluctuation for the electromagnetic and the muonic component. We also report a complete study of the parameters that are affected by the thinning and semi-analytical estimations of this fluctuation.**

***Keywords*: Fluctuation simulation thinning**

I. Introduction

Extensive air showers (EAS) have been studied over the last 70 years [1]. They result from the interaction in the atmosphere of high-energy particles arriving from space. The products of these collisions are a set of secondary particles sharing the energy of the primary particle initiating the shower. These secondaries move through the atmosphere and interact again generating new secondaries. The process continues and the number of particles increases, until their energies are too low to continue with the particle generation. In this cascading phenomena, the complexity and the poor knowledge of the hadronic interactions at very high energy make the study of the development of the shower quite difficult. The secondary particles can be detected with ground based detectors and correlated with the primary particle. Measurements of the electron and muon density, the arrival time of the particles, and the depth at which the shower has the maximum number of particles provides valuable information on the primary energy, arrival direction and mass composition of the primary particle. The physical interpretation of these observables is however not easy because even primary particles with the same energy, composition and direction produce secondaries having characteristics that vary from shower to shower. This feature is called the *"shower to shower fluctuations"*. An understanding of the shower to shower fluctuations will help to improve the measurement of the composition of the primaries and refine the measurements.

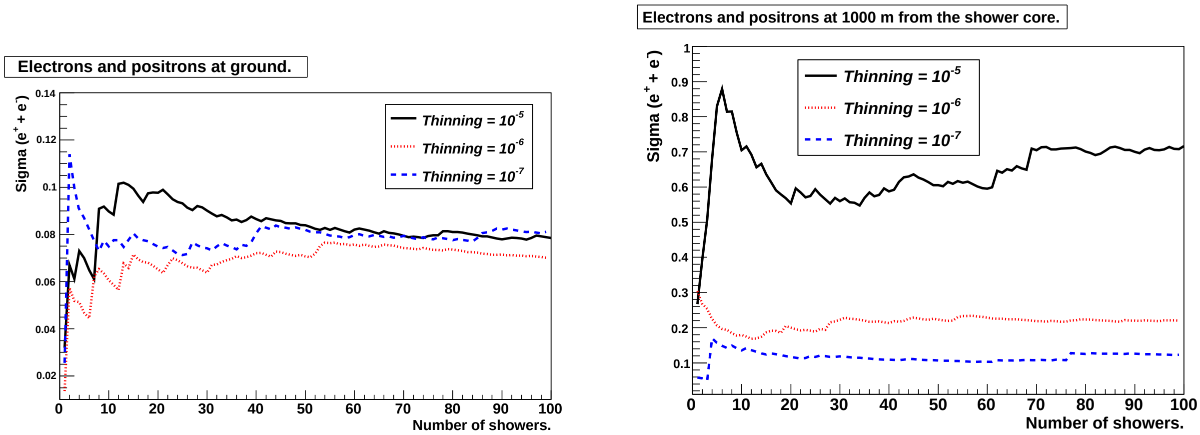
Besides this, in the case of simulated data, the number of particles that are produced in an EAS at ultra high energy is too large as to make impossible the propagation of all the secondaries. For instance, the total number of particles in an EAS initiated by a 10 EeV proton is $\sim 10^{11}$, being almost impossible to store all the necessary data for such amount of particles. EAS simulations at the highest energies are only possible because they use a statistical sampling algorithm called thinning. Thinning consists on propagating only a small fraction of the total number of particles in the shower and to assign statistical weights to the sampled particles to compensate for the rejected ones. However that thinning procedure introduces artificial fluctuations in the parameters of the showers. For this reason the study of fluctuations using Monte Carlo simulations is quite difficult because they are strongly affected by the thinning in a not very well controlled manner. An incorrect assumption on the fluctuation in the shower observables can lead to systematic uncertainties on their measurement.

In this work we study the shower to shower fluctuations using Monte Carlo simulations of shower development. In particular we study how the thinning algorithm affects the calculation of the fluctuations and we present a method to approximately separate the fluctuations introduced by thinning from those due to the intrinsic fluctuations of the physical processes that occur in the shower. This paper is organized as follows: In section II we describe the shower simulations and the thinning algorithm. In section III we describe the fluctuations and classify them. Our results are reported in section IV. We summarize our conclusions in section V.

II. Extensive air shower simulations

We have simulated EAS using the Aires code [2], [3] which makes use of a thinning algorithm [5], in which the weights assigned to the sampled particles are adjusted in such a way that both the total energy and the average number of particles are guaranteed to be conserved.

Before the simulation starts the user sets the relative thinning level $R_{\rm th}$. The thinning energy $E_{\rm th}$ – the energy below which the thinning algorithm is applied – is obtained as $E_{\rm th}$=$R_{\rm th} \times E_p$, where E_p is the energy of the primary particle initiating the shower. For ultra high energy cosmic ray shower simulations convenient values for the relative thinning are $R_{\rm th} = 10^{-5} - 10^{-9}$, the actual choice depends on the calculation to be performed. The $R_{\rm th}$ chosen affects both the simulation CPU time and the size of its output, both typically behaving linearly with $R_{\rm th}^{-1}$. If for instance $R_{\rm th}$ is increased by a factor of 10 the simulation speeds up and the size



(a) (b)

Fig. 1. Relative RMS ($\sigma/\text{Average}$) of the total number of electrons (left) in the ground (left) and in a ring at $r = 1000$ m around the shower axis (right) versus the shower number. The black solid line corresponds to simulation using thinning level $R_{\text{th}} = 10^{-5}$. The red dotted line corresponds to simulations with $R_{\text{th}} = 10^{-6}$. The blue dashed line corresponds to simulation with $R_{\text{th}} = 10^{-7}$.

of the output file is reduced by about the same factor, but the price to pay is an enhancement of the artifical fluctuations in the simulated showers (see more details about the thinning algorithm in the Aires Manual [3]).

In Aires, the thinning algorithm is further complemented with an "extended thinning algorithm" which was designed to guarantee that the statistical weights are always smaller than a certain positive number. To ensure this, an external parameter, called statistical weight factor can be controlled by the user. To optimize the procedure of sampling, different weight factors for electromagnetic ($W(\text{EM})$) and heavy particles ($W(\text{HADRONIC})$) are defined. The parameter $W(\text{HADRONIC})$ is specified indirectly through the ratio (see [3])

$$\text{AEH} = \frac{W(\text{EM})}{W(\text{HADRONIC})}. \qquad (1)$$

The default value of this ratio in the simulation is $\text{AEH} = 88$. The default value of the weight factor $W(\text{EM})$ is 12. In this paper, we will use these default values unless otherwise indicated.

We have simulated proton and iron showers with primary energy 10 EeV, zenith angle 0° and relative thinning 10^{-5}, 10^{-6} and 10^{-7} (10^{-7} for proton showers only). Simulations at other zenith angles and energies have also been performed. To study the fluctuations due to the depth of the first interaction point (see below), we have also simulated showers with fixed first interaction depth.

III. Classification of Fluctuations

Rather generally we can make a simple classification of fluctuations in shower development:

- *Physical fluctuations:* Due to fluctuations in the depth, multiplicity, inelasticity, etc... of the first interaction and of the secondary interactions.
- *Experimental fluctuations:* Fluctuations due to the response of the detector, sampling fluctuations, etc.
- *Artificial fluctuations:* Induced by the thinning and un-thinning (re-sampling) processes.

The "physical fluctuations" can be further splitted into those due to the first interaction and those induced in the secondary interactions, as is customary. This splitting is mainly motivated by the fact that "universal" shower properties may emerge when accounting for the fluctuations in the first interaction point [6]. Here we refer to *"fluctuations in the first interaction"* as those due to the fluctuation in the depth where the first interaction occurs only, keeping in mind that other fluctuations arise in the first interaction, such as those induced by multiplicity and inelasticity which we do not include under this definition.

In the case of real showers, fluctuations are enlarged due to the response of the detector and to the fact that the detector typically samples a small fraction of the whole shower. The detector response introduces an additional source of fluctuations, which are detector dependent and will not be considered in this work. The sampling fluctuation is a statistically well-known problem, and will not be considered further here.

On the other hand, simulated showers are affected by artificial fluctuations due to the thinning and un-thinning procedures. These affect the interpretation of Monte Carlo simulations and make them difficult to be usable in the generation of artificial events. In this work we will not consider the effect of unthinning [7].

IV. Results

In Fig. 1 we show the relative RMS ($\sigma/\text{average}$) of the total number of electrons at ground and for electrons falling in a ring at a distance r =1000 m from the shower core. In both cases the relative RMS is plotted as a function of the number of showers simulated. The ring was taken from r_{min} =912 m to r_{max} =1092 m where the radii correspond to a logarithmically symmetric interval around 1000 m to compensate for the steep decrease of the density of particles with r. It can be seen that the fluctuations in the ring around 1000 m are larger than the fluctuations when the whole ground is considered, and also that the fluctuations in the ring have a stronger dependence with the thinning level $R_{\rm th}$ used. This is easy to understand. If we consider an entry (a sampled particle in the shower) of weight w falling inside the ring, it would be equivalent to w particles entering inside the same ring, so that if due to a physical fluctuation we lose or gain that entry, we would also lose or gain w particles inside the ring, and the fluctuations are enlarged. The plots also clearly display that no reliable evaluation of the RMS can be done with less than about 20 simulated showers, especially for fluctuations in the ring. Also a thinning factor $R_{\rm th} > 10^{-5}$ is not sufficient to evaluate the RMS.

We have further studied the particle fluctuations in a ring around the shower axis. For this purpose we define:

- $\bar{w}$: The average weight of entries (sampled particles in the shower) falling in the ring.
- Ω: The RMS of the distribution of weights of the entries.
- $\bar{N}_e$: The average number of entries (sampled particles).
- s: The RMS of the number of entries.
- $\bar{N}$: Average total number of particles (note that $\bar{N} = \bar{w}\bar{N}_e$).
- σ: The RMS of the total number of particles.

and with the aid of our simulations we have studied their behaviour with the thinning level and with the distance to the shower core.

The average weight $\bar{w}$ for electrons and muons is found to be simply proportional to the thinning level $\bar{w} \propto R_{\rm th}$, as expected. In the case of the electromagnetic component $\bar{w}$ decreases with r because at large distances from the shower core the electrons are mainly produced by muon decay which typically carry a smaller weight. In the case of the muonic component higher energy muons are typically produced closer to the shower axis and typically carry smaller weights, while lower energy muons arrive far from the core and carry larger weigths. The relative RMS of the weight $\Omega/\bar{w}$ is found to be almost independent of $R_{\rm th}$ or equivalently $\Omega \propto R_{\rm th}$ for small values of $R_{\rm th}$. The average number of entries is found to be $\bar{N}_e \propto R_{\rm th}^{-1}$ as expected since the thinning algorithm is made in such a way that $\bar{N} = \bar{w}\bar{N}_e$ (so that the average number of particles is independent $R_{\rm th}$). Finally for $s/\bar{N}_e$ we obtain that the relative fluctuation is approximately independent of the thinning level, implying that $s \propto R_{\rm th}^{-1}$.

Using these parameters it is possible to obtain an analitycal expresion for the fluctuations in the total number of particles. It can be proven that:

$$\sigma^2 = \sigma_{\rm th}^2 + \sigma_{\rm phys}^2 = \bar{N}_e\Omega^2 + \bar{w}^2 s^2. \qquad (2)$$

The proof only assumes that the probability for a sampled particle to have a given weight is independent of the probability of a shower to have a given number of sampled particles [8]. In Eq. (2) $\sigma_{\rm phys}$ contains the fluctuations due to physical processes in the shower, while $\sigma_{\rm th}$ contains the fluctuations associated to the thinning procedure [9].

Each of the two terms in the formula for σ^2 has in fact a different behaviour with the thinning level. Since $\bar{N}_e \propto R_{\rm th}^{-1}$ and $\Omega \propto R_{\rm th}$ the fluctuations induced by thinning behave as:

$$\sigma_{\rm th}^2 \propto R_{\rm th}, \qquad (3)$$

– as expected for fluctuations due to thinning. Also since $\bar{w} \propto R_{\rm th}$ and $s \propto R_{\rm th}^{-1}$ as obtained above, we have:

$$\sigma_{\rm phys}^2 \sim {\rm constant}. \qquad (4)$$

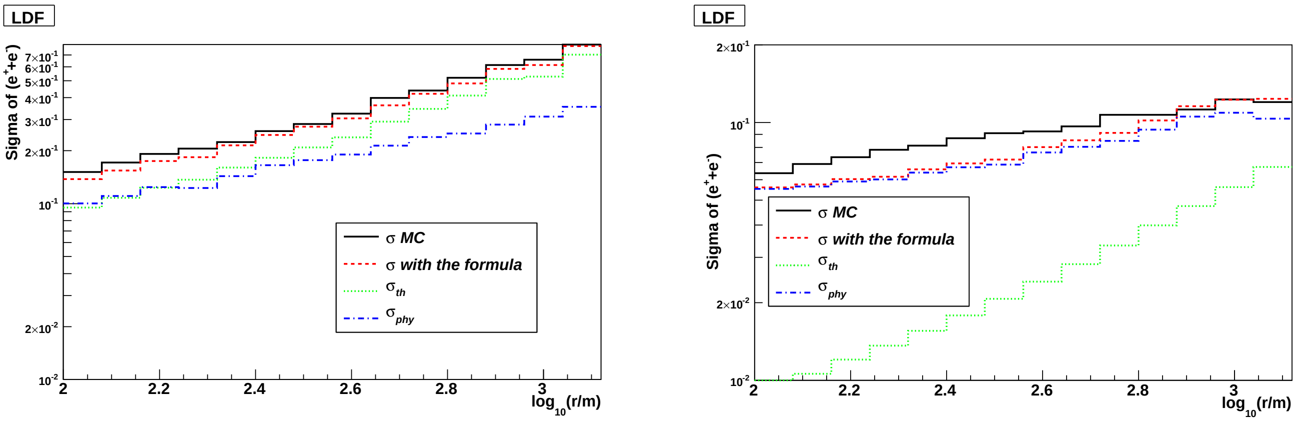
i.e. the fluctuations associated to physical processes do not depend on the thinning level. This was of course expected but it is remarkable that it was obtained from the Monte Carlo simulation, and therefore gives a strong support to our identification of $\sigma_{\rm phys}$ with the true physical fluctuations. Moreover, if $R_{\rm th} \to 0$ (when full Monte Carlo simulations of shower development are performed) clearly $\Omega \to 0$ and the fluctuations are only due to the second term in Eq. (2), i.e. they are the physical fluctuations. In this case $\bar{w} \to 1$, $\bar{N}_e \to \bar{N}$ and $\sigma \to s$ as expected.

In Figs. 2 and 3 we compare the fluctuations predicted by Eq. (2) with those obtained in the simulation. A good agreement is seen for muons and electrons in 10 EeV proton showers simulated with two relative thinning levels of $R_{\rm th} = 10^{-5}$ and 10^{-7}. We also plot the contribution of $\sigma_{\rm th}$ and σ_{phys} to the total σ as predicted by Eq. (2). It can be seen that for $R_{\rm th} = 10^{-5}$ the fluctuations due to thinning are a significant fraction of the total fluctuations, however for $R_{\rm th} = 10^{-7}$ these are very much reduced as expected, and are in fact much smaller than the physical fluctuations.

Finally, we have also studied the importance of the fluctuation in the first interaction depth. For this purpose we have performed two types of simulations: Regular simulations in which the depth of the first interaction point is allowed to vary, and simulations in which we have fixed the first interaction depth of the primary particle (proton or iron) to the mean interaction depth of that primary in the QGSJET [4] model (for proton the mean free path is ~ 45 g cm^{-2} at 10 EeV and for iron ~ 10.7 g cm^2). The result is that the fluctuations in both simulations are of comparable value, which clearly shows that we can not see the fluctuation due to the first interaction, because it is obscured by the much larger artificial (thinning) fluctuations at least for $R_{\rm th} > 10^{-7}$.

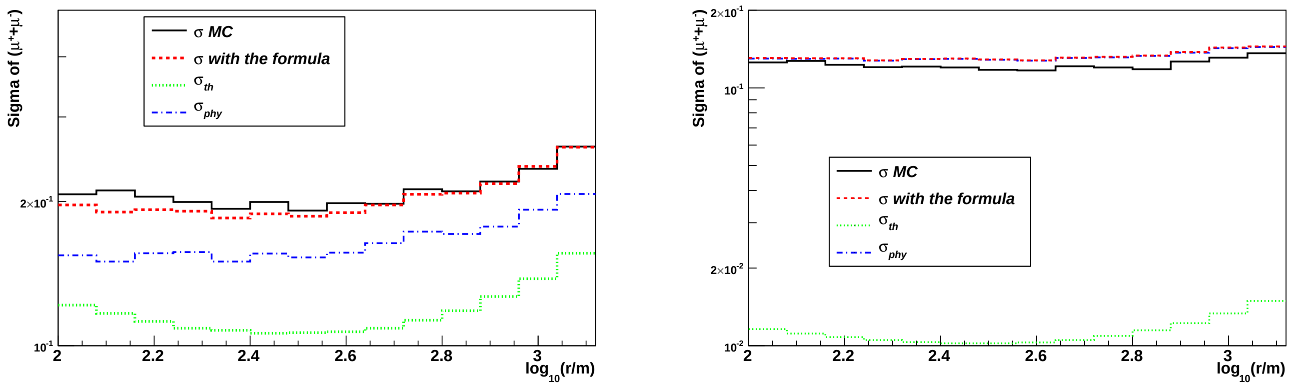
V. CONCLUSIONS

We study how the artificial fluctuations introduced by thinning affect the calculation of the physical shower to shower fluctuations. We have seen that for thinning



(a) (b)

Fig. 2. Left panel: Fluctuations (σ) of the total number of electrons at ground as obtained in Monte Carlo simulations of 10 EeV proton showers with $R_{\rm th} = 10^{-5}$ (black solid line), and fluctuations predicted by Eq. (2) (red dashed line). Also shown are the contributions to the fluctuations due to thinning $\sigma_{\rm th}$ (green dotted line) and that due to physical processes in the shower $\sigma_{\rm phys}$ (blue dash-dotted line) as predicted by Eq. (2). Right panel: Same as in left panel for 10 EeV proton showers with $R_{\rm th} = 10^{-7}$.



(a) (b)

Fig. 3. Same as in Fig. 2 for the total number of muons.

levels $R_{\rm th} = 10^{-5} - 10^{-6}$, the determination of shower to shower fluctuation is hampered by the artificial fluctuations introduced by the thinning procedure.

We have obtained a semi-analytical formula which gives a good description of the fluctuations in the total number of particles, and which in an approximate manner allows to separate the physical shower to shower fluctuations from the artificial fluctuations induced by thinning. Moreover Eq. (2) allows to obtain the physical shower to shower fluctuations from Monte Carlo simulations for an arbitrary thinning level smaller than $R_{\rm th} \sim 10^{-5}$.

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