Cosmic Ray Perpendicular Diffusion and Acceleration at an Oblique Shock

Andreas Shalchi*, **Gang Li**† **and Gary P. Zank**†

*Theoretische Physik IV, Ruhr-Universität Bochum, Germany
†CSPAR, University of Alabama, Huntsville, USA*

Abstract. **We investigate cosmic ray scattering in the direction perpendicular to a mean magnetic field. Unlike in previous articles we employ a general form of the turbulence wave spectrum with arbitrary behavior in the energy range. By using an improved version of the nonlinear guiding center theory we compute analytically the perpendicular mean free path. As shown the energy range spectral index has a strong influence on the perpendicular diffusion coefficient. Using this new analytical formula, we examine the problem of particle acceleration at an perpendicular interplanetary shock.**

***Keywords*: Cosmic Rays, Turbulence, Solar System**

I. INTRODUCTION

The propagation of cosmic rays in the solar system and in the interstellar medium continues to be a subject of great interest (see, e.g., Matthaeus et al. 2003, Shalchi et al. 2004b, Shalchi & Schlickeiser 2005, Shalchi et al. 2006, Zimbardo et al. 2006, Shalchi & Kourakis 2007) as is particle acceleration at interplanetary shocks, such as those driven by coronal mass ejections, or in supernova remnants (see, e.g, Bell 1978a,b, Zank et al. 2000, Zank et al. 2006, Berezhko & Völk 2007). For these problems one of the most important elements entering the fundamental equations is the cosmic ray diffusion tensor.

A theory for cross-field diffusion is the so-called nonlinear guiding center (NLGC) theory (Matthaeus et al. 2003). In the following we combine the improved version of this theory proposed by Shalchi & Dosch (2008) with a general wave spectrum in the energy range. We compute analytically the perpendicular mean free path by using methods developed earlier (see Shalchi et al. 2004a). These results are than applied on diffusive shock acceleration at an perpendicular interplanetary shock. Details can be found in Shalchi et al. (2009).

II. THE GENERAL SPECTRUM AND DOMINANT TWO-DIMENSIONAL MODES

By superposing slab and two-dimensional fluctuations, a quasi three-dimensional model for approximating solar wind turbulence can be formulated. This model is often called a two-component or slab/2D composite model of magnetic turbulence. In addition to this turbulent field we take into account a non-vanishing mean magnetic field $\vec{B}_0 =< \vec{B} >$ which may be identified with the magnetic field of the Sun. For two-component turbulence we find for the magnetic correlation tensor in the wave vector space $P_{lm}(\vec{k}) =< \delta B_l(\vec{k}) \delta B_m^*(\vec{k}) >= P_{lm}^{slab}(\vec{k}) + P_{lm}^{2D}(\vec{k})$ with

TABLE I
TURBULENCE PARAMETERS.

Parameter	Physical meaning
s	Inertial range spectral index
q	Energy range spectral index
$D(s,q)$	Normalization function for general $q > -1$
l_{slab}	Slab bendover scale
l_{2D}	2D bendover scale
δB_{slab}^2	Magnetic energy of the slab fluctuations
δB_{2D}^2	Magnetic energy of the 2D fluctuations

$$P_{lm}^{slab}(\vec{k}) = g^{slab}(k_\parallel) \frac{\delta(k_\perp)}{k_\perp} \delta_{lm}, \tag{1}$$

for the slab modes, and

$$P_{lm}^{2D}(\vec{k}) = g^{2D}(k_\perp) \frac{\delta(k_\parallel)}{k_\perp} \left[\delta_{lm} - \frac{k_l k_m}{k^2} \right], \tag{2}$$

for the two-dimensional modes. Note that we have used $l, m = x, y$ and that tensor components with z are zero. In Eqs. (1) and (2) we have used the wave spectra of the slab modes $g^{slab}(k_\parallel)$ and of the two-dimensional modes $g^{2D}(k_\perp)$.

In the following, we assume that the two-dimensional contribution to the perpendicular diffusion coefficient is dominant. Therefore we approximate the correlation function by the tensor of the two-dimensional modes of Eq. (2).

For the spectrum we use the form proposed by Shalchi & Weinhorst (2009)

$$\begin{aligned} g^{2D}(k_\perp) &= \frac{2D(s,q)}{\pi} \delta B_{2D}^2 l_{2D} \\ &\times \frac{(k_\perp l_{2D})^q}{[1 + (k_\perp l_{2D})^2]^{(s+q)/2}}. \end{aligned} \tag{3}$$

The spectrum $g^{2D}(k_\perp)$ has to fulfil the normalization constraint $\delta B^2 = \delta B_x^2 + \delta B_y^2 = \int d^3k \ \ [P_{xx} + P_{yy}]$ (where we have used $P_{zz} = 0$) resulting in $D(s,q) = \Gamma\left[(s+q)/2\right] / \left\{2\Gamma\left[(s-1)/2\right]\Gamma\left[(q+1)/2\right]\right\}$ where $\Gamma(z)$ is the Gamma-function. The parameters used in the spectrum are listed in Table I. We depict the spectrum in Fig. 1 for different values of q. The spectrum is correctly normalized for $s > 1$ and $q > -1$.

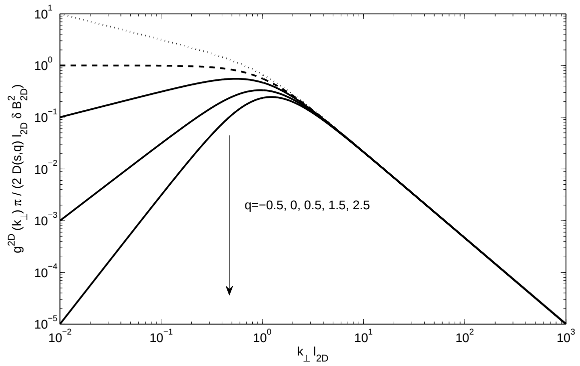

Fig. 1. The turbulence spectrum of the two-dimensional modes for different values of the energy range spectral index q. For $q = 0$ we have a constant spectrum, for $q < 0$ a decreasing spectrum, and for $q > 0$ an increasing spectrum in the energy range.

III. Theoretical description of perpendicular diffusion

A. The improved theory of Shalchi & Dosch

The original nonlinear guiding theory of Matthaeus et al. (2003) is based on the equation

$$\tilde{v}_x = a v_z \frac{\delta B_x}{B_0} \tag{4}$$

where a is a parameter related to the guiding center approximation that cannot be derived from first principles. Therefore, Eq. (4) was replaced by Shalchi & Dosch (2008) by an (exact) integral representation of the Newton-Lorentz equation. These authors found the following integral equation

$$\kappa_\perp = \frac{v^2}{3B_0^2} \int d^3k \, \frac{P_{xx}(\vec{k})}{\alpha(\vec{k})} \beta(\vec{k}) \tag{5}$$

for the perpendicular diffusion coefficient $\kappa_\perp$. Here we have used

$$\alpha(\vec{k}) = \gamma(\vec{k}) + \frac{v}{\lambda_\parallel} + \sum_{n=x,y,z} \kappa_n k_n^2. \tag{6}$$

and

$$\beta(\vec{k}) = \frac{2\alpha^2(\vec{k}) + \Omega^2}{\alpha^2(\vec{k}) + \Omega^2}. \tag{7}$$

In Eqs. (5)-(7) we have used the unperturbed gyrofrequency $\Omega := qB_0/(\gamma mc)$ with the Lorentz-factor γ, the particle velocity v, the particle charge q, the particle mass m, the speed of light c, the spatial diffusion coefficients κ_n, and the parallel mean free path $\lambda_\parallel = 3\kappa_\parallel/v$. The parameter $\gamma(\vec{k})$ is the inverse dynamical correlation time of the turbulence. Since we concentrate on high energy particles, dynamical turbulence effects should not be important and, therefore, we can employ the magnetostatic approximation by setting $\gamma(\vec{k}) = 0$.

B. Analytical forms of $\kappa_\perp$

Eq. (5) can be simplified by noting that the parameter β defined in Eq. (7) satisfies $1 \leq \beta \leq 2$. Therefore we can approximate Eq. (5) by

$$\kappa_\perp = a^2 \frac{v^2}{3B_0^2} \int d^3k \, \frac{P_{xx}(\vec{k})}{\alpha(\vec{k})} \tag{8}$$

where the parameter a^2 satisfies $1 \leq a^2 \leq 2$. As shown by Shalchi & Dosch (2008), we find $a^2 \approx 1$ for low energy particles and $a^2 \approx 2$ for higher particle energies. The first limit corresponds to the case that the charged particles follow magnetic field lines. Eq. (8) is in agreement with the formula originally derived in Matthaeus et al. (2003).

For pure two-dimensional modes we use Eq. (2) to obtain

$$\kappa_\perp = a^2 \frac{\pi v^2}{3B_0^2} \int_0^\infty dk_\perp \, \frac{g^{2D}(k_\perp)}{v/\lambda_\parallel + \kappa_\perp k_\perp^2}. \tag{9}$$

With the wave spectrum from Eq. (3) and the methods presented in Shalchi et al. (2004a) we can solve Eq. (9) analytically (for the mathematical details see Shalchi et al. 2009).

We find for $\lambda_\parallel \lambda_\perp \gg 3 l_{2D}^2$ and $q < 1$

$$\begin{aligned} \lambda_\perp &= \left[3^{(q+1)/2} a^2 D(s,q) \frac{\delta B_{2D}^2}{B_0^2} l_{2D}^{q+1} \right]^{2/(q+3)} \\ &\times \left[\Gamma\left(\frac{1+q}{2}\right) \Gamma\left(\frac{1-q}{2}\right) \right]^{2/(q+3)} \\ &\times \lambda_\parallel^{(1-q)/(3+q)}. \end{aligned} \tag{10}$$

For $\lambda_\parallel \lambda_\perp \gg 3 l_{2D}^2$ and $q > 1$ the solution of Eq. (9) is

$$\lambda_\perp = \sqrt{\frac{3(s-1)}{2(q-1)}} a l_{2D} \frac{\delta B_{2D}}{B_0} \tag{11}$$

and for $\lambda_\parallel \lambda_\perp \ll 3 l_{2D}^2$ and arbitrary q we can derive

$$\lambda_\perp = \frac{a^2}{2} \frac{\delta B_{2D}^2}{B_0^2} \lambda_\parallel. \tag{12}$$

For $\lambda_\parallel \lambda_\perp \gg 3 l_{2D}^2$ and $q < 1$, we find a complicated relation between the mean free paths in different directions. For $q > 1$ the perpendicular mean free path becomes independent of the parallel mean free path and is, therefore, energy independent. For $\lambda_\parallel \lambda_\perp \ll 3 l_{2D}^2$ we find that the perpendicular mean free path is directly proportional to the parallel mean free path and independent of q.

C. The influence of the parameter q

In Fig. 2 we have calculated the perpendicular mean free path numerically from Eq. (9) for different values of the energy range spectral index q. Clearly we find for $q > 1$ that the perpendicular mean free path becomes independent of the parallel mean free path for large values of $\lambda_\parallel$. Small values of q lead to a much larger parallel mean free path. Obviously the parameter q has a strong influence on the perpendicular mean free path.

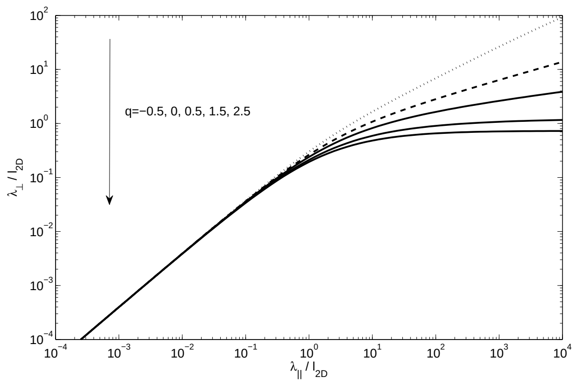

Fig. 2. The perpendicular mean free path versus the parallel mean free path for different values of q. Negative values of q (dotted line) correspond to a descreasing spectrum in the energy range, $q = 0$ (dashed line) corresponds to the constant spectrum used previously, and positive values of q (solid lines) correspond to an increasing spectrum.

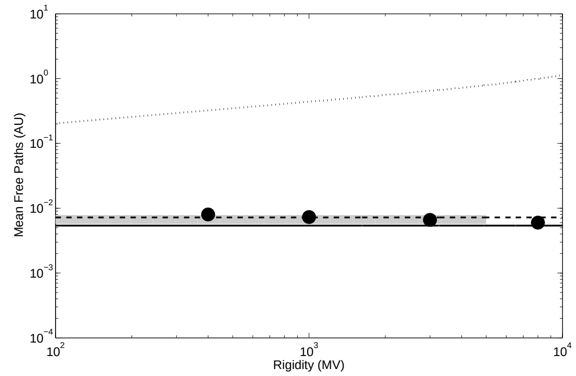

Fig. 3. The perpendicular mean free path $\lambda_\perp$. Shown are Ulysses measurements of Galactic protons (Burger et al. 2000, dots) in comparison with the analytical results of Eq. (11) for $l_{2D} = 0.003AU$ (solid line) and for $l_{2D} = 0.0039AU$ (dashed line). Also shown is the value for the perpendicular mean free path as suggested by Palmer (1982, grey line). For comparison also the quasilinear parallel mean free path is shown (dotted line).

IV. Application I: Transport of Galactic protons in the solar system

Here we investigate perpendicular scattering for high energy particles in the solar system. Burger et al. (2000) derived from Ulysses measurements of Galactic protons that $\lambda_\perp \approx 0.006 - 0.008$ AU at higher rigidities. Palmer (1982) concluded an average perpendicular diffusion coefficient at 1 AU of $\kappa_\perp \ c/v \approx 10^{21} \ cm^2/s$ corresponding to $\lambda_\perp \approx 0.0067AU$ (in this paper the author pointed out that the spread around this average was rather large). These two observational results are shown in Fig. 3. For $q = 1.5$ (in agreement with Matthaeus et al. 2007), $s = 5/3$ (Kolmogorov spectrum), $\delta B_{2D}^2 / B_0^2 = 0.8$ (see Bieber et al. 1996), and $a^2 = 2$ (as derived by Shalchi & Dosch 2008 for high energy particles) we find from Eq. (11): $\lambda_\perp \approx 1.8 l_{2D}$. The perpendicular mean free path is directly proportional to the bendover scale of the two-dimensional modes l_{2D}. This characteristic length scale has so far not been determined by observations. In previous articles (see, e.g., Matthaeus et al. 2003, Shalchi et al. 2006) it has been assumed that $l_{2D} \approx 0.003AU$ leading to $\lambda_\perp \approx 0.0054AU$ which is a factor 1.3 smaller than the Burger et al. (2000) value. The best agreement with the Ulysses measurement can be obtained by setting $l_{2D} \approx 0.0039AU$ which cannot be excluded by observations.

V. Application II: Particle acceleration at highly perpendicular interplanetary shock waves

A mechanism that yields the high energies of cosmic rays is diffusive shock acceleration (see, e.g., Axford et al. 1977, Bell 1978a,b, Blandford & Ostriker 1978). In the following we revisit the problem of particle acceleration at perpendicular interplanetary shock waves. Examples of such interplanetary shocks are those driven by coronal mass ejections (CME's). The theory of particle acceleration at quasi-parallel shocks appears to be reasonably well understood, and has been applied to solar energetic particle and energetic storm particle events (see, e.g., Zank et al. 2000, Li et al. 2003, 2005). Zank et al. (2006) have developed a theory for describing particle acceleration at quasi-perpendicular shocks. In this case the characteristic acceleration timescale is given by (see, e.g., Webb et al. 1995)

$$\tau_{acc} = \left(\frac{1}{p} \frac{dp}{dt} \right)^{-1} = \frac{3\sigma}{v_{sh}^2 (\sigma - 1)} \kappa_\perp(p), \qquad (13)$$

where the shock velocity v_{sh} and the compression ratio $\sigma = v_{up}/v_{down}$ have been introduced (here we used the upstream and downstream velocities of the shock). By integrating Eq. (13) we find for the dynamical time scale $t = r/\dot{r}$ of the propagating shock

$$t = \frac{3\sigma}{v_{sh}^2(\sigma - 1)} \int_{p_{inj}}^{p_{max}} dp \, \frac{\kappa_\perp(p)}{p}. \qquad (14)$$

This formula yields the time that is needed to accelerate a particle from an injection momentum p_{inj} to a maximum momentum p_{max}. The relation between particle velocity and momentum is $p = mv\gamma$ with the Lorentz-factor γ. We can rewrite Eq. (14) by replacing the diffusion coefficient by the mean free path to find

$$t = \frac{\sigma}{v_{sh}^2(\sigma - 1)} \int_{v_{inj}}^{v_{max}} dv \, \gamma^2 \lambda_\perp(v). \qquad (15)$$

By using the dimensionless time

$$\tau = \frac{t v_{sh}^2}{\lambda_\perp c} \frac{\sigma - 1}{\sigma} \qquad (16)$$

and by employing Eq. (11) corresponding to $\lambda_\perp(v) = \lambda_\perp = const.$, we derive

$$\frac{v_{max}}{c} = tanh(\tau), \qquad (17)$$

$$\frac{p_{max}}{mc} = sinh(\tau), \qquad (18)$$

and for the maximum (relativistic) kinetic energy E_{max}

$$\frac{E_{max}}{mc^2} = \cosh(\tau) - 1. \quad (19)$$

For mathematical details we refer again to Shalchi et al. (2009). To evaluate these formulas we replace the time t in Eq. (16) by the ratio of shock position r and shock velocity $\dot{r}$: $t = r/\dot{r}$. For the perpendicular mean free path we use Eq. (11) and, therefore,

$$\lambda_{\perp} = \lambda_{\perp,0} \frac{\delta B_{2D}}{B_0} \quad (20)$$

with

$$\lambda_{\perp,0} = \sqrt{\frac{3(s-1)}{2(q-1)}} a l_{2D}. \quad (21)$$

Thus, we obtain

$$\tau = \frac{r}{\dot{r}} \frac{v_{sh}^2}{\lambda_{\perp,0} c} \frac{\sigma - 1}{\sigma} \frac{B_0}{\delta B_{2D}}. \quad (22)$$

For the standard parameters ($s = 5/3$, $q = 1.5$, $a = \sqrt{2}$, and $l_{2D} = 0.003AU$) we have $\lambda_{\perp,0} = 0.006AU$.

To proceed we have to estimate the shock parameters r, $\dot{r}$, v_{sh}, and σ as well as the ratio $B_0/\delta B_{2D}$. For a strong shock it is reasonable to assume that $\dot{r} = v_{sh}$. Furthermore, Shalchi et al. (2009) have derived

$$\frac{B_0}{\delta B_{2D}} \approx \sqrt{\frac{1AU}{r} + \frac{r}{1AU}} \quad (23)$$

for the magnetic fields. To obtain this formula a simple WKB model for the turbulence and a Parker spiral model (Parker 1958) for the mean field was employed. As also shown in Shalchi et al. (2009) we find for a Sedov-Taylor blast wave solution that

$$\frac{\dot{r}}{c} \approx 4 \cdot 10^{-3} \left(\frac{1AU}{r} \right)^{3/2}. \quad (24)$$

$\lambda_{\perp,0}$ can be approximated by $\lambda_{\perp,0} = 0.006AU$ and for the compression ratio we use $\sigma = 3.7$. By combining Eqs. (16)-(24) with these values we can derive

$$\tau = 0.5 \sqrt{\frac{r}{1AU} + \left(\frac{r}{1AU} \right)^3} \left(\frac{1AU}{r} \right)^{3/2}. \quad (25)$$

The parameter τ can easily be combined with Eqs. (17), (18), and (19) to compute the maximum velocity, momentum, or kinetic energy. The maximum kinetic energy a proton can get by interacting with a CME driven shock is illustrated in Fig. 4. For small heliocentric distances we have $E_{max} \approx 1.3$GeV and for $r \approx 1$AU we find $E_{max} \approx 200$MeV. This result is consistent with that derived originally in Zank et al. (2000) who also obtained GeV energies for protons during the early phase of shock wave propagation. Both here and in Zank et al. (2000), the reason for the high energies is the combination of the large magnetic field strength close to the Sun and the speed/strength of the shock. The decay in the maximum particle energy accelerated at the shock is a consequence of the IMF strength weakening with increasing heliocentric distance, as well as the slowing down of the interplanetary shock.

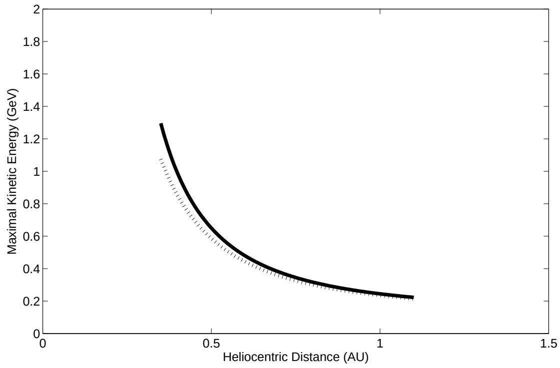

Fig. 4. The maximum kinetic energy E_{max} that a proton can achieve due to diffusive shock acceleration as a function of heliocentric distance. Here we have employed the analytical result obtained for the Sedov-Taylor phase of the shock. Shown is the correct relativistic result (solid line) as well as the non-relativistic approximation (dotted line).

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