

Modeling a relation between the shower age and lateral distribution parameters of charged particles in EAS

A.A. Ivanov, M.I. Pravdin and A.V. Sabourov

Shafer Institute for Cosmophysical Research and Aeronomy, Yakutsk 677980, Russia

Abstract. The lateral distribution function (LDF) of particles in extensive air shower (EAS) of cosmic rays (CRs) evolves along the cascade propagating in the atmosphere. There is a universal relation between LDF parameters and the shower age which can be used to infer the maximum depth, X_{max} , of EAS from LDF measured with the surface arrays. We have simulated the relation by the Monte Carlo method implemented in CORSIKA code for hadronic interaction models and primary CR particles. Our aim is to specify an algorithm of the shower age and/or X_{max} estimation applicable to the Yakutsk array data, in particular.

Keywords: Extensive air shower age

I. INTRODUCTION

In 1958 Nishimura and Kamata have solved Landau's equations for the lateral distribution of electrons and photons in approximation B of the cascade theory [1]. The solution evaluated in Moliere units, R_M , appears to be a function of the shower age, s , characterizing the cascade development. It is obvious in Greisen's approximation of Nishimura and Kamata solution (well-known NKG-function):

$$f(R/R_M) \propto \left(\frac{R}{R_M}\right)^{s-2} \left(1 + \frac{R}{R_M}\right)^{s-4.5}.$$

In extensive air shower there are hadrons and muons in addition to electrons and photons, so the NKG-function can't be applied directly; the lateral distribution shape is a function of energy and zenith angle, at least. However, there may be a shower core distance interval for a given primary energy, E_0 , where one can neglect the contribution of hadrons and muons to the measured charged particles density; and where the LDF shape, approximately, is a function of the shower age only.

We have simulated showers using CORSIKA code, with different primary nuclei and photons initiating EAS, in order to demonstrate that the lateral distribution parameters of charged particles in the core distance interval $100 < R < 1000$ m (energy $E_0 > 1$ EeV, zenith angle $\theta < 50^\circ$) are functions of the shower age only, in a good approximation. In other words, the LDF shape dependence on EAS energy, zenith angle and even on the primary particle type is parameterized by the shower age.

In this paper, we have used two parameters connected with the LDF shape: RMS radius and the slope of radial distribution, although other variables can be used as well.

The shower age is approximated as $s = 3X_L/(X_L + 2X_{max})$, where X_L, X_{max} are the observer and the shower maximum slant depths in the atmosphere.

II. PARAMETRIZATION OF ANGULAR AND ENERGY DEPENDENCIES OF LDF

Root mean square (RMS) radius of LDF is calculated in a ring $100 < R < 1000$ m:

$$R_{RMS}^2 = \frac{1}{N} \int_{100}^{1000} R^2 f(R) 2\pi R dR,$$

where N is a number of particles in the ring.

The LDF slope, b , is evaluated in the same ring using least square method with the weights proportional to the inverse square of relative deviations of particle densities. These parameters: RMS radius and LDF slope are connected with each other via the common LDF shape. It is evident if to use the same approximation of the lateral distribution to calculate R_{RMS} and b . For instance, in the simplest case when the power dependence on radius, $f(R) = cR^{-b}$, is applicable:

$$\langle R^2 \rangle = \frac{2 - b}{4 - b} \frac{R_2^{4-b} - R_1^{4-b}}{R_2^{2-b} - R_1^{2-b}},$$

where $R_1 = 100$ m; $R_2 = 1000$ m.

Artificial showers are simulated with CORSIKA code, version 6.710 [2], where the hadronic interaction model SIBYLL, version 2.1 [3], is implemented. Low energy interactions are modeled with UrQMD, version 1.3.1 [4]. A set of primary particle energies $E_0 \in (1, 10)$ EeV and types: γ, P, C, Fe are used to calculate the shower age and LDF at sea level; a thinning level is 10^{-5} ; zenith angle is chosen within $\theta \in (0^\circ, 50^\circ)$; the Moliere radius is set for Yakutsk winter conditions $R_M = 66.9$ m. The azimuthal dependence of the shower parameters is neglected.

The average RMS radius of charged particles LDF is shown in Fig. 1, left panel, as a function of the shower age. In the right panel a slope dependence on the age is shown. Two panels demonstrate a parametrization of the LDF shape by the shower age: all the energies, zenith angles and primary particles used in samples result in RMS radii and LDF slopes belonging to the same universal function, in a good approximation. Of course, this result concerns the average values only; fluctuations disperse the dependence, as shown by RMS deviations. Functions are slightly different for primary nuclei, more difference is seen for primary photons; but all the differences are less than 5%.

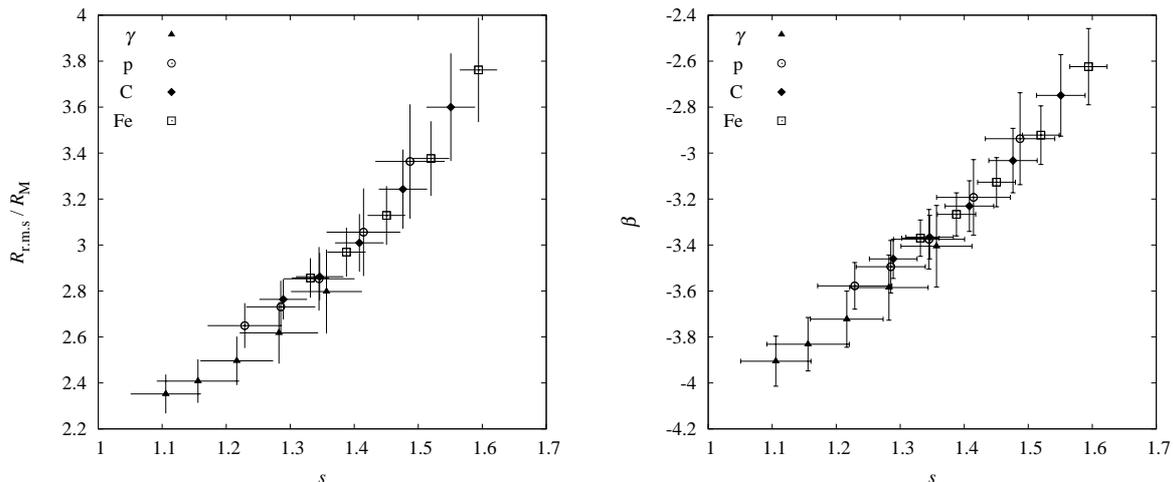


Fig. 1: Root mean square radius (in Moliere units) and slope of LDF at sea level as a function of the shower age. Horizontal bars are RMS deviations of the age in samples; vertical bars are deviations of the RMS radius and slope of LDF.

We are planning to apply the procedure to other hadronic interaction models implemented in CORSIKA: QGSJETII, EPOS; other types of primary particles are of interest, too. The Yakutsk array arrangement is convenient to measure LDF slope, rather than the RMS radius. So we will focus on this parameter in the rest of the paper.

It is interesting to note that, as a sequence of considerations above, the energy dependence of LDF slope is connected to X_{max} :

$$\begin{aligned} \frac{\partial b}{\partial \lg E_0} &= \frac{db}{ds} \left(\frac{\partial s}{\partial X_{max}} \frac{dX_{max}}{d \lg E_0} + \frac{\partial s}{\partial \cos \theta} \frac{d \cos \theta}{d \lg E_0} \right) = \\ &= \frac{\partial b}{\partial X_{max}} \frac{dX_{max}}{d \lg E_0} = ER \frac{\partial b}{\partial X_{max}}, \end{aligned}$$

where $ER = dX_{max}/d \lg E_0$ is a so-called 'Elongation Rate', X_{max} shift with the primary energy. Measuring $\partial b/\partial \lg E_0$ we can estimate $\partial b/\partial X_{max}$ basing on ER value given by PAO/HiRes experiments [5], [6].

Another possibility to estimate $\partial b/\partial X_{max}$ is to measure zenith angle dependence of the LDF slope and application of the relation (see Appendix):

$$X_{max} \frac{\partial b}{\partial X_{max}} = \cos \theta \frac{\partial b}{\partial \cos \theta}.$$

In logarithms, the relation is quite simple:

$$\frac{\partial b}{\partial \log X_{max}} = \frac{\partial b}{\partial \log \cos \theta}.$$

There is no need for ER in this case!

III. AGE DEPENDENCE OF LDF MEASURED IN YAKUTSK

An additional benefit of findings above is that we don't need X_{max} measurement in order to connect, say, LDF slope with the primary particle mass of EAS. Measuring its zenith angle dependence, for instance, we

simultaneously get energy/ X_{max} dependencies due to b being a function of the age.

We are going to apply a procedure to the data of the Yakutsk array, fixing E_0 in narrow bins, setting radial distance and zenith angle limits as described in the first section. As it is readily apparent from Linsley's 'Elongation Rate theorem' [7], X_{max} is independent of zenith angle. So all the showers in a sample we select have the same maximum slant depth¹; the age variance is a consequence of the atmospheric thickness being a function of zenith angle $X_L = X_0 \sec \theta$; $X_0 = 1020 \text{ g/cm}^2$.

Our enthusiasm is based on the preliminary results of the LDF slope measurement with the Yakutsk array given in Fig. 2. The slope measured is a monotonous function of $\sec \theta$ and E_0 . A comparison of the slope derivatives with respect to zenith angle and primary energy with simulation results can allow us to choose the particular hadronic interaction model or the primary UHECR composition compatible with the experimental data. The derivatives may differ in models due to various X_{max} and ER in spite of the same $b(s)$ relation.

As a first step, the derivative $\partial b/\partial \sec \theta$ is estimated at fixed energies in comparison with the values calculated using SIBYLL model with different primary particles initiating EAS. Results are shown in Fig. 3. Due to rather large uncertainties of the experimental points it is impossible to choose the particular primary particle type with this comparison alone. May be, the results of EAS induced by photons are somewhat below the experimental values. There is a need for further refined analysis of more parameters to make definite conclusions.

¹ignoring E_0 variance within the energy bin

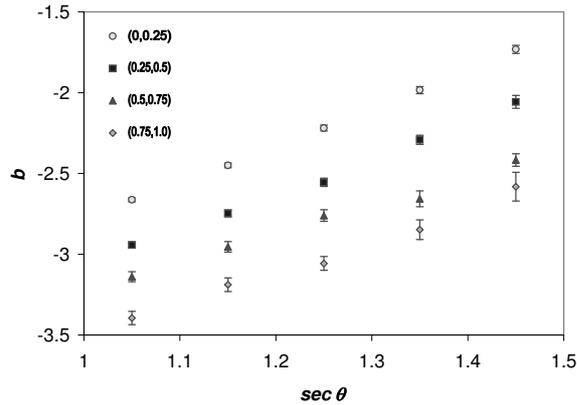


Fig. 2: LDF slopes measured in Yakutsk. Logarithm of primary energy, EeV, is within intervals given on the left. Average b estimation errors in samples are shown by the vertical bars.

IV. CONCLUSIONS

Modeling EAS with CORSIKA code using different primary particles but the same hadronic interaction model (SIBYLL) we have demonstrated that the RMS radius and slope of the lateral distribution of charged particles at sea level are the universal functions of the shower age. This feature concerns the average values only in the limited intervals of the shower core distance and zenith angle.

Simulated shower parameters can be compared with experimental data even without reference to the independent measurements of X_{max} or ER . The derivative $\partial b / \partial \sec \theta$ is compared with the data of the Yakutsk array, but the results are indefinite due to experimental errors, as yet. There is a need for further refined analysis of more parameters to make definite conclusions.

Previously, there were speculations (e.g. [8], [9]) that the LDF slope is related to X_{max} in the hadronic model-independent manner. Instead, we have proved that the LDF slope² is a universal function of the shower age. X_{max} dependence can be derived when the primary energy or zenith angle are fixed.

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²as well as the RMS radius

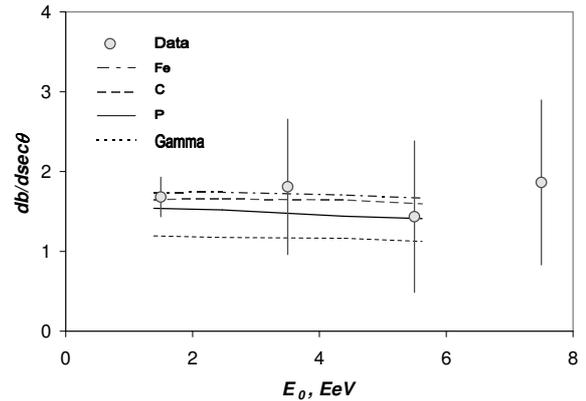


Fig. 3: Comparison of experimental and simulated values of $\partial b / \partial \sec \theta$. Expected values for EAS primary particles calculated with CORSIKA code are given by curves. Derivatives in narrow energy bins are estimated using the Yakutsk array data.

APPENDIX

DEALING WITH DERIVATIVES

A distinctive feature of the LDF slope we assumed is $b(s)$. The derivatives with respect to X_{max} and $\cos \theta$ are:

$$\frac{\partial b}{\partial X_{max}} = \frac{db}{ds} \frac{\partial s}{\partial X_{max}} = -\frac{db}{ds} \frac{6 \cos \theta}{X_0(1 + 2X_{max} \cos \theta / X_0)^2}$$

$$\frac{\partial b}{\partial \cos \theta} = \frac{db}{ds} \frac{\partial s}{\partial \cos \theta} = -\frac{db}{ds} \frac{6X_{max}}{X_0(1 + 2X_{max} \cos \theta / X_0)^2}.$$

Multipliers X_{max} and $\cos \theta$ reduce it to the identity:

$$X_{max} \frac{\partial b}{\partial X_{max}} = \cos \theta \frac{\partial b}{\partial \cos \theta}.$$

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